6:5 Positive and Negative Numbers

Teacher Notes

Central math concepts

This task is about disentangling notions of order from notions of magnitude. A rational number has a location on the number line, and it also has a magnitude or absolute value, which is the number's distance from 0 on the number line. For rational numbers *r* and *s*, the inequality *r* > *s* is a statement of order: *r* > *s* means that *r* is to the right of *s* on the number line. For example, 4 > 2 and 0 > -1.

Why is it necessary to disentangle notions of order from notions of magnitude? Consider the following two statements about two numbers *r* and *s* on the number line:

r is to the right of *s*.

r is farther from 0 than *s*.

If we restrict *r* and *s* to the nonnegative numbers, then these two statements would be equivalent (see figure, top). But for rational numbers, the two statements are not equivalent. For example, 1 is to the right of -2, but -2 is farther from 0 than 1 (see figure, bottom).

In elementary grades, the symbol - has long been used to indicate subtraction, as in the problem 7 - 2 = 5. The rational numbers introduce an additional use for this symbol, as in the example of the number -8. Here, the - sign is not indicating an operation on two numbers; rather, -8 refers to the additive inverse of 8 (the number that makes 0 when added to 8). In grade 6, prior to learning how to operate with signed numbers, the number -8 can be understood in various contexts as "the opposite of 8" (for example, if the number 8 in context represents a location 8 feet above sea level, then the number -8 represents a location 8 feet below sea level). The number -8 can be understood on the number line as the mirror reflection of the number 8 across the 0 point. Reflecting then reflecting again leaves a number unchanged, just as the opposite of the opposite of "up" is "up." In the context of grade 7 operations with rational numbers, the number -8 is fundamentally the number that makes 0 when added to 8, also the result of 0 - 8 and the result of (-1)·(8).

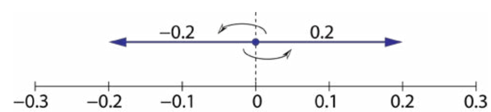
Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: number sense of fraction size; comparing fractions; locating points on a number line; and basing mathematical explanations on a number line diagram.

6:5 **(1)** Which of the numbers $5, -7, \frac{2}{3}, -\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$. **(3)** Explain why $-(-0.2) = 0.2$ makes sense.

Answer

(1) -7 is farthest from 0, and $-\frac{1}{2}$ is closest to 0. **(2)** True. **(3)** Answers may vary, may be written or spoken, and may be supported by such things as diagrams or evocations of contexts. For example, "The opposite of the opposite of a number is just the number," or "Reflecting twice takes you back to where you started." For an example of a diagram, see the figure, in which 0.2 and -0.2 are represented by arrows based at 0, with length 0.2, and direction indicated by the sign of the number.



Starting with 0.2, -0.2 is the reflection of that, and -(-0.2) is the reflection of that. Which is 0.2. 'Reflecting twice takes you back to where you started.'

Click here for a student-facing version of the task.

Refer to the Standards

6.NS.C.6, 7; MP.3, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Extending the task

How might students drive the conversation further?

- Students could create contexts that help make sense of parts (1) and (2) of the task. For example, if A has $5 and B owes $7, then does B owe more than A has? Is $\frac{1}{2}$ degree Fahrenheit above zero a warmer temperature than 8 degrees Fahrenheit below zero?
- Students could generalize the reasoning of part (3) to pose puzzle problems such as -(-(-8))) = ?. What if there were more than three minus signs?

Related Math Milestones tasks

6:3

6:3 The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

Time	Hours after Midnight	Temp °F
8:00 pm	−4	−42
9:00 pm	−3	−42
10:00 pm	−2	−41
11:00 pm	−1	−40
Midnight	0	−39
1:00 am	1	−39
2:00 am	2	−38

Plot the data on graph paper and label the plot. Describe any patterns you see.

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$.

(2) Show calculation 1(a) on a number line.

7:12

7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

7:5

7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

5:10

5:10 **(1)** Solve: $\frac{1}{3} = 0.1 + ?$

(2) Is there a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$? If you think so, find such a number. If you think there is no such number, explain why.

(3) Show one of the above problems and its solution on a number line.

Task **6:3 South Pole Temperatures** involves a set of time series data that uses negative values to represent times before midnight.

In later grades, task **7:9 Calculating with Rational Numbers** involves the four operations on rational numbers, and task **7:12 Temperature Change** involves arithmetic with signed rational numbers in context. Task **7:5 Is There a Solution? (Addition)** focuses on an equation that has no solution in the positive numbers but that can be solved in the rational number system.

In earlier grades, task **5:10 Number System, Number Line** explores the structure of the positive number line.

Additional notes on the design of the task

- The varied numbers in the task seek to portray positive and negative fractions and decimals as an integrated system of numbers.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:5? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 6:5? In what specific ways do they differ from 6:5?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:5 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?