6:6 Planting Corn

Teacher Notes

Math Milestones™

Central math concepts

A farmer plants 216 acres every 12 hours. How many acres does the farmer plant in 6 hours?

In 2 hours?

In 3 hours?

In 18 hours?

We could solve a succession of such problems individually, and indeed part (1) of task **6:4 Gas Mileage** includes several problems along these lines—but the more such problems we solve, the more we may begin to suspect that we're repeating ourselves. Isn't there some single, essential fact about this farmer's situation that we should try to put our finger on?

The key to the farmer's situation is to think about what happens in 1 hour. In 1 hour, the farmer plants 216 ÷ 12 = 18 acres. 18 acres per hour is the *unit rate* for the proportional relationship between the number of acres and the number of hours. Why is the unit rate helpful? To see why, first set aside the unit rate and consider again that problem of how many acres the farmer plants in 6 hours. A solution might be based on the fact that because 6 hours is half of 12 hours, the farmer will plant half of 216 acres in 6 hours: in other words, 108 acres. In finding that solution, we have used the given information (216 acres, 12 hours) together with a scaling argument: in half the time, the farmer finishes half the work. But what if the given information had been more useful? What if, instead of specifying that the farmer plants 216 acres every 12 hours, the problem had said, "A farmer plants 18 acres every 1 hour." Then the problem of how many acres are planted in 6 hours would certainly be no harder: in 6 times the time, the farmer finishes 6 times the work, or 6 × 18 = 108 acres. That's the same answer we obtained before.

The advantage of the unit rate becomes clearer if we consider a question like, "How many acres are planted in 5 hours?" Using the originally given information, we could certainly say that in $\frac{5}{12}$ of the time, $\frac{5}{12}$ of the work gets done: $\frac{5}{12} \times 216$ acres are planted. Equally well, we could set up a proportion and solve it: $\frac{5}{12} = \frac{A}{216}$. However, the better information that "a farmer plants 18 acres every 1 hour" is so helpful, we might as well just use that information directly and argue that in 5 times as many hours, 5 times as much work gets done: therefore 5 × 18 = 90 acres are planted. The unit rate is essentially an improved version of the originally given information. The problem didn't originally provide the unit rate to us, but we are still entitled to provide it to ourselves.

Knowing that 18 acres are planted every hour, we can find the number of acres planted in any number of hours, *n*. That's because, in *n* times as many hours, *n* times as much work gets done:

Number of acres planted in *n* hours = *n* times the number of acres planted in 1 hour

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in *n* hours.

Answer

$A = 18n$ (students might use a different letter for the acreage variable or use different conventions to indicate multiplication such as $18 \times n$ or $18 \cdot n$ or 18 * *n*).

Click here for a student-facing version of the task.

Refer to the Standards

6.RP.A, 6.EE.C.9; MP.2, MP.4, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- The numbers 216 and 12 were chosen because of the many divisibilities they offer. This not only makes the unit rate a whole number, but it also creates the possibility of students creating an informative table using only whole-number values.
- A farmer in this newspaper article planted 312 acres in a 16-hour day, which is a rate of 19.5 acres per hour.

Number of acres planted in n hours = n × unit rate

Number of acres planted in n hours = n × 18.

To reach this conclusion, we didn't have to "set up a proportion." But for the sake of seeing the problem from many sides, let's now approach the problem that way. In doing so, we can use the variables A and n to stand for the number of acres and the number of hours. Then the proportion is the equation,

$$\frac{216}{12} = \frac{A}{n}.$$

Instead of cross-multiplying right away, let's first complete the division on the left-hand side:

$$18 = \frac{A}{n}.$$

If we *now* cross-multiply, then we obtain the answer to the task: $18n = A$.

The equation $18 = \frac{A}{n}$ is a more informative statement about the situation than the proportion $\frac{216}{12} = \frac{A}{n}$, because $18 = \frac{A}{n}$ shows the value of the unit rate.

Setting up and solving a proportion only ever gives a single number as the result. But task 6:6 is about the functional thinking inherent in a proportional relationship. That's why the answer to the problem is a formula, not a single number.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: mental calculation; using ideas of scaling and times-as-much; and defining and using a variable.

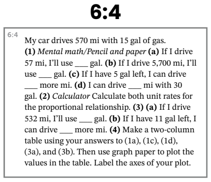
Extending the task

How might students drive the conversation further?

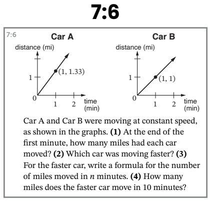
- Students can use the equation $A = 18n$ to determine how many hours or days it would take to plant 1,000 or 10,000 acres of corn.
- How many hours would be spent planting 90 million acres of corn, which is about how many acres are planted annually in the United States?
- If a farmer works a 12-hour day during planting season, then how many days would that much planting take? If planting season lasts only about a month, then how many farmers are needed to plant all that corn?

Related Math Milestones tasks

6:4

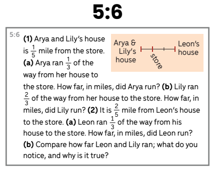

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

7:6



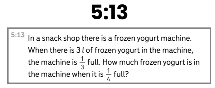
Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

5:6



5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

5:13


5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

Another Math Milestones task for grade 6 that prominently features a proportional relationship is **6:4 Gas Mileage**. Whereas in task 6:6 a function

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:6? Locate 2–3 similar tasks in that unit. How are the tasks similar to each other, and to 6:6? In what specific ways do they differ from 6:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

equation expresses the relationship, in task 6:4 a table and a graph are used to analyze the relationship.

In later grades, task **7:6 Car A and Car B** prominently features a proportional relationship and the idea of unit rate, with a graphical representation as central.

In earlier grades, tasks **5:6 Corner Store** and **5:13 Frozen Yogurt Machine** situate the multiplicative scaling idea of times-as-much in context.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?