6:8 Evaluating an Expression

Teacher Notes

Math Milestones™

Central math concepts

Working with expressions involves looking for and making use of structure. Just as the expression 4 × 76 × 25 rewards pausing before diving in and multiplying the factors in given order from left to right, the expression in task 6:8 rewards pausing before diving in and substituting into the given expression. And just as rewriting 4 × 76 × 25 in the equivalent form 4 × 25 × 76 serves a purpose of making a calculation easier, combining like terms in task 6:8 serves the same purpose. Finally, just as rewriting 4 × 76 × 25 as 4 × 25 × 76 relies on properties of operations (associativity and commutativity of multiplication), collecting like terms in task 6:8 also relies on properties of operations, in particular the distributive property:

$$
\begin{aligned}
& 0.96r + 0.04r - r \\
&= 0.96r + 0.04r - 1r \\
&= (0.96 + 0.04 - 1)r \\
&= (1 - 1)r \\
&= 0.
\end{aligned}
$$

This shows that the expression will evaluate to 0 no matter what value of r is substituted into it.

There isn't a standard algorithm for evaluating or transforming algebraic expressions; instead, there are choices to make. Those choices require comprehension of the structure of expressions, as well as linguistic fluency with the syntax of expressions and their conventions (such as omitting the multiplication sign, using the same symbol for subtraction as for negation, or understanding that a term "r" has a coefficient of 1). As examples like 4 × 76 × 25 or 4,999 + 12 illustrate, calculation in the elementary grades was never only algorithmic,[†] and in the middle grades and high school it seldom ever is.[‡]

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: adding two-digit decimals; interpreting written conventions of algebraic expressions; using the distributive property; and viewing expressions as objects with structure.

Extending the task

How might students drive the conversation further?

- Students could see what happens if 1 is substituted for r in the given expression. (Is substituting 1 a kind of "trick" for collecting like terms?)
- Suppose the second coefficient in the given expression is changed from 0.04 to 0.05. What will be the result of substituting r = 11,000?

> 6:8 *Pencils down* If $r = 1.748$, what is the value of $0.96r + 0.04r - r$?

Answer

0.

Click here for a student-facing version of the task.

Refer to the Standards

6.EE.A; MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency

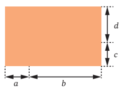
Additional notes on the design of the task

- The combination of "pencils down" together with the use of decimals to thousandths (1.748) is intended to create an incentive to look for a labor-saving approach. However, the purpose of task 6:8 isn't to differentiate between students who do or don't think of collecting like terms. Rather, the purpose is to help all students see the power of looking at the structure of an expression and seeing what opportunities it affords.
- The first two terms in the expression might make sense and/or lead to ideas if interpreted as, "96% of something plus 4% of that thing."

Related Math Milestones tasks

6:11

6:11 The diagram shows a rectangle. The variables a, b, c, and d are lengths in meters.



(1) Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

Task **6:11 Area Expressions** involves transforming expressions into equivalent forms using properties of operations. **Task 6:13 Is There a Solution? (Multiplication)** is a task that, like 6:8, surfaces concepts in a topic that involves procedural fluency.

7:1

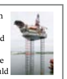
7:1 The cost of a phone is the phone's price, $264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1-1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$.
(2) Show calculation 1(a) on a number line.

7:8

7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?



8:2

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

In later grades, tasks **7:1 Phone Cost**, **7:9 Calculating with Rational Numbers**, and **7:8 Oil Business** also involve the distributive property. Task **8:2 Pottery Factory** involves a situation that can be modeled with an equation in which properties of operations including the distributive property would be used in the process of solving for an unknown value.

3:2

3:2

The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.

Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.

12×14, 11×14, 12×15, 11×15

4:5

4:5 **(1a–f)** Write the values of the products. Compare answers with a classmate.

$4 \times \frac{1}{7} =$ ___ (a)
$6 \times \frac{4}{7} =$ ___ (b)
$86 \times \frac{1}{86} =$ ___ (c)
$6 \times \frac{8}{2} =$ ___ (d)
$9 \times \frac{1}{9} =$ ___ (e)
$9 \times \frac{2}{9} =$ ___ (f)

(1g) Which answer is twice as much as the answer for (e)?
(1h) Which answer is six times as much as the answer for (a)?
(1i) Which two answers are equal?
(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

5:14

5:14 Brandon was reading his math book. He saw the equation $\frac{3}{4} \times (4 + \frac{1}{2}) = 3 + \frac{3}{8}$. He said, "I don't get it—where did the 3 and the $\frac{3}{8}$ come from?" Write an explanation that could answer Brandon's question.

5:1

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

In earlier grades, tasks **3:2 Hidden Rug Design**, **4:5 Fraction Products and Properties**, and **5:14 Brandon's Multiplication Equation** are tasks involving expressions as objects with structure. Task **5:1 Juice Box Mixup** has an interpretation in terms of multiplication distributing over subtraction.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:8? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:8? In what specific ways do they differ from 6:8?

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:8 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?[3]

† National Research Council. 2001. Adding It Up: Helping Children Learn Mathematics. Washington, DC: The National Academies Press. Page 121.

‡ William McCallum (2008), "Mindful Manipulation: What Algebra Do Students Need for Calculus?" (presentation)

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?