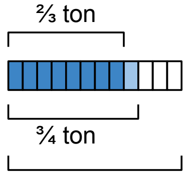
6:9 Truckload of Gravel

Teacher Notes

Central math concepts

Because $\frac{2}{3}$ is less than $\frac{3}{4}$, the answer to task 6:9 must be a fraction less than 1. An analogous problem posed with whole numbers might read, "How much of a 9-liter container is 8 liters of water?" The solution to that problem is the unknown factor in $\square \times 9 = 8$, which is the quotient $8 \div 9$. Likewise, the solution to task 6:9 is the unknown factor in $\square \times \frac{3}{4} = \frac{2}{3}$, which is the quotient $\frac{2}{3} \div \frac{3}{4}$. Students used such unknown-factor reasoning in earlier grades, for example when they thought along the lines of, "7 times something is 42, so the something must be $42 \div 7$."

Fraction equivalence and unit fractions play a role, visible or invisible, in most fraction situations. For example, suppose we replace the fractions $\frac{3}{4}$ and $\frac{2}{3}$ in task 6:9 by equivalent fractions with the same denominator, $\frac{9}{12}$ and $\frac{8}{12}$. Think of $\frac{1}{12}$ ton as a new unit of gravel, a "twelfth-ton" of gravel. Then the question is equivalent to, "How much of a 9-unit quantity is 8 units?" This may suggest the answer $\frac{8}{9}$ or the process $8 \div 9$. Thinking about twelfths of a ton may also help to create a diagram of the situation. In this diagram, $\frac{2}{3}$ ton is shown as 8 parts in a partition of $\frac{3}{4}$ ton into 9 parts.



Algebraically, the structure of task 6:9 corresponds to the equation $F \times \frac{3}{4} = \frac{2}{3}$, where F is the desired fraction. Dividing both sides of the equation by $\frac{3}{4}$ produces the unknown factor: $F = \frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$.

Calculating the quotient $\frac{2}{3} \div \frac{3}{4}$ is equivalent to calculating the product $\frac{2}{3} \times \frac{4}{3}$. This circumstance is connected to a second way to solve the equation $F \times \frac{3}{4} = \frac{2}{3}$, which is to multiply both sides of the equation by $\frac{4}{3}$:

$$\frac{4}{3} \times \left(F \times \frac{3}{4}\right) = \frac{4}{3} \times \frac{2}{3}$$
$$\frac{4}{3} \times \left(\frac{3}{4} \times F\right) = \frac{4}{3} \times \frac{2}{3}$$
$$\left(\frac{4}{3} \times \frac{3}{4}\right) \times F = \frac{4}{3} \times \frac{2}{3}$$
$$1 \times F = \frac{8}{9}$$
$$F = \frac{8}{9}.$$

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

Answer

$\frac{2}{3}$ ton of gravel is $\frac{8}{9}$ of a $\frac{3}{4}$-ton truckload of gravel.

Click here for a student-facing version of the task.

Refer to the Standards

6.NS.A.1, 6.EE.B.7; MP.2, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- Task 6:9 isn't well thought of as a proportional relationships problem. The context doesn't feature two variable quantities, just a single unknown value. The equation, $F \cdot \frac{3}{4} = \frac{2}{3}$ is a constraint equation, not a function equation. That said, the observation in Central Math Concepts about scaling up shows that fraction quotients and fraction equivalence can sometimes involve versions of proportional thinking.

This solution approach relates to the context in the following way. Imagine scaling up both of the quantities in task 6:9 by a factor of four-thirds. Because $\frac{4}{3} \times \frac{3}{4} = 1$ and $\frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$, the scaled-up problem will say:

> How much of a 1-ton truckload of gravel is $\frac{8}{9}$ ton of gravel?

The answer to this question, $\frac{8}{9}$, is plain, and this answer must also be the answer to the original question in task 6:9, because both quantities were scaled by the same factor. It is as if we have changed the units in the problem from truckloads measuring $\frac{3}{4}$ ton to more convenient units of "big-truckloads" measuring 1 ton.

For an illuminating discussion of fraction division with numerous diagrams and examples, see the 2017 series of blog posts by William McCallum and Kristin Umland on MathematicalMusings.org (part 1, part 2, part 3, part 4).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: thinking about unit fractions in context; using ideas of scaling and times-as-much; and defining a variable to create and solve a constraint equation.

Extending the task

How might students drive the conversation further?

- Students who used different approaches or who drew mathematically different diagrams to represent the problem could explain their thinking to each other.
- Students could discuss whether it makes sense that the answer, $\frac{8}{9}$, is less than 1.

Related Math Milestones tasks

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

6:2

6:2 **(1)** Would you prefer 33% of a $100 prize or 75% of a $50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a $16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a $100 item adds ___ dollars to the cost. A 3% tax on a $1 item adds ___ dollars to the cost.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

6:14

6:14 *Pencil and paper* **(1)** $81.53 \div 3.1 = ?$ **(2)** $\frac{7}{8} \div \frac{2}{3} = ?$ **(3)** Check both of your answers by multiplying.

Like task 6:9, task **6:1 Charging Cord** is a word problem with an unknown factor that is a quotient of fractions. Task **6:13 Is There a Solution? (Multiplication)** deals with the existence and nature of solutions of a particular equation with form $ax = b$. Task **6:2 Prizes, Prices, and Percents** includes percent problems with the mathematical structure of an unknown factor problem (parts (2) and (3)). Task **6:4 Gas Mileage** includes several scaling problems in the context of a single proportional relationship. Task **6:14 Dividing Decimals and Fractions** is a procedural task that involves dividing fractions.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:9? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 6:9? In what specific ways do they differ from 6:9?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:9 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

In later grades, task **7:13 Wire Circle** is a word problem that could be solved by creating a constraint equation of the form $px + q = r$, where p, q, r, and x are all non-whole numbers.

5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

5:1

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

5:5

5:5 Write the requested values.

$4087 \times 53 = ?$	$\frac{1}{10} \div 10 = ?$	$0.4 \times 0.9 = ?$
$246 \times 914 = ?$	$\frac{7}{8} \times \frac{5}{3} = ?$	$0.75 \div 0.01 = ?$
$9744 \div 12 = ?$	$8 \times ? = 73$	$0.63 \div 0.3 = ?$
$1461 \div 6 = ?$	$3 \div \frac{1}{8} = ?$	$0.86 + 0.4 = ?$
$4 - (8 - 4) = ?$	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	$0.72 - 0.17 = ?$
	$\frac{1}{3} \div (6 \times 5) = ?$	$0.02 + 0.2 = ?$
		$0.8 - 0.55 = ?$
		$637 - 1.31 = ?$

In earlier grades, task **5:13 Frozen Yogurt Machine** is a multi-step word problem involving multiplication and division with unit fractions. Task **5:1 Juice Box Mixup** involves an unknown factor that is a fractional quotient of whole numbers. Task **5:5 Calculating** is a procedural task that includes quotients involving whole numbers and unit fractions.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?