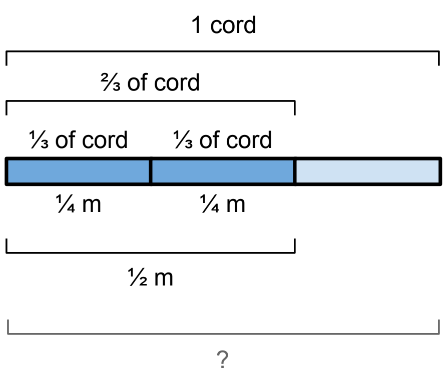
6:1 Charging Cord

Teacher Notes

Central math concepts

Unit fractions play a role, visible or invisible, in most fraction situations. For example, suppose we knew that $\frac{1}{3}$ of a cup of flour weighs $4\frac{1}{4}$ ounces. Then we would know that 1 cup of flour weighs 3 times as much. Or, suppose we knew that in a newspaper survey, $\frac{1}{10}$ of the people surveyed, or 85 people, preferred a certain monthly subscription plan. Then we would know that 850 people were surveyed. Even when a problem is given without unit fractions, thinking about unit fractions in the problem can help. For example, if $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ m long, then how long in meters is $\frac{1}{3}$ of the cord? Half as long, or $\frac{1}{4}$ m. The realization that $\frac{1}{3}$ of the cord is $\frac{1}{4}$ m long may be helpful in determining the full length of the cord. (See the diagram.)



Fraction equivalence also plays a visible or invisible role in many fraction problems. For example, if we replace the fraction $\frac{1}{2}$ in task 6:1 with the equivalent fraction $\frac{2}{4}$, then the original statement that $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ m long becomes the equivalent statement that **2 thirds** of a charging cord is **2 fourths** of a meter long. Thirds of the cord match up with fourths of a meter, as shown in the diagram. So 1 cord is 3 thirds which is 3 fourths of a meter.

Algebraically, the structure of task 6:1 corresponds to the equation $\frac{2}{3}L = \frac{1}{2}$, where L is the length of the cord in meters. Dividing both sides of the equation by $\frac{2}{3}$ produces the length of the cord: $L = \frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$. This process could correspond to an understanding of division along the lines of, "$\frac{2}{3}$ times something is $\frac{1}{2}$...so the something must be $\frac{1}{2} \div \frac{2}{3}$." This is a generalization of the reasoning that students used in grade 3, when they

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

Answer

$\frac{3}{4}$ m. (The decimal 0.75 m is also correct.)

Click here for a student-facing version of the task.

Refer to the Standards

6.NS.A.1, 6.EE.B.7; MP.2, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- Task 6:1 could be stated in equivalent terms as, "$\frac{2}{3}$ of a charging cord is 50 centimeters long. How long is the cord?" The structure of this form of the problem is $\frac{2}{3}C = 50$, where C is the length of the cord in centimeters. In this form, the problem continues to reward thinking along the lines discussed in the Central Math Concepts. For example, dividing both sides of the equation by $\frac{2}{3}$ gives $C = 50 \div \frac{2}{3} = 50 \times \frac{3}{2} = 75$; and thinking that $\frac{1}{3}$ of the cord is 25 cm long may suggest that the cord is 3 × 25 cm long. However, when providing the answer to the task, the length of the cord must be expressed in meters.

solved problems like “7 times something is 42, so the something must be 42 ÷ 7.”

Calculating the quotient $\frac{1}{2} \div \frac{2}{3}$ is equivalent to calculating the product $\frac{1}{2} \times \frac{3}{2}$. This circumstance is connected to a second way to solve the equation $\frac{2}{3}L = \frac{1}{2}$, which is to multiply both sides of the equation by $\frac{3}{2}$:

$$\frac{3}{2}\left(\frac{2}{3}L\right) = \frac{3}{2}\left(\frac{1}{2}\right)$$

$$\left(\frac{3}{2}\cdot\frac{2}{3}\right)L = \frac{3}{2}\left(\frac{1}{2}\right)$$

$$(1)L = \frac{3}{4}$$

$$L = \frac{3}{4}.$$

Multiplying both sides of the equation by $\frac{3}{2}$ corresponds to the idea that “I could turn $\frac{2}{3}$ into 1 whole by multiplying it by one-and-a-half, so the length of the cord must be one-and-a-half times $\frac{1}{2}$ m.”

For an illuminating discussion of fraction division with numerous diagrams and examples, see the 2017 series of blog posts by William McCallum and Kristin Umland on MathematicalMusings.org (part 1, part 2, part 3, part 4).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: thinking about unit fractions in context; using ideas of scaling and times-as-much; and defining a variable to create and solve a constraint equation.

Extending the task

How might students drive the conversation further?

- Students who worked with units of centimeters and students who worked with units of meters could relate the two approaches by creating a combined diagram that aligns the centimeter diagram and the meter diagram.
- Students could discuss whether it makes sense that the cord is less than 1 meter long, in light of the fact that more than half of the cord is only half of a meter.

Additional notes on the design of the task (continued)

- Because length is a continuous quantity, length quantities can be good candidates for fraction contexts. Length has special importance because length is the basic metaphor for all measured magnitudes.
- Task 6:1 isn’t well thought of as a proportional relationships problem. The context doesn’t feature two variable quantities, just a single unknown value. Thus, $\frac{2}{3}L = \frac{1}{2}$ or $\frac{2}{3}C = 50$ are constraint equations, not function equations. As in previous grades, the division operation here simply finds an unknown factor; it doesn’t yield a third kind of quantity, as when we divide distance by time to create a new quantity speed, or divide mass by volume to create a new quantity density. (That said, fraction quotients and fraction equivalence can sometimes involve versions of proportional thinking, as for example when we think, “If the parts are twice as many, each part must be half the size.”)

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:1? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 6:1? In what specific ways do they differ from 6:1?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Related Math Milestones tasks

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

6:2

6:2 **(1)** Would you prefer 33% of a $100 prize or 75% of a $50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a $16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a $100 item adds ___ dollars to the cost. A 3% tax on a $1 item adds ___ dollars to the cost.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

6:14

6:14 *Pencil and paper* **(1)** $81.53 \div 3.1 = ?$ **(2)** $\frac{7}{8} \div \frac{2}{3} = ?$ **(3)** Check both of your answers by multiplying.

Like task 6:1, task **6:9 Truckload of Gravel** is a word problem with an unknown factor that is a quotient of fractions. Task **6:13 Is There a Solution? (Multiplication)** deals with the existence and nature of solutions of a particular equation with form $ax = b$. Task **6:2 Prizes, Prices, and Percents** includes percent problems with the mathematical structure of an unknown factor problem (parts (2) and (3)). Task **6:4 Gas Mileage** includes several scaling problems in the context of a single proportional relationship. Task **6:14 Dividing Decimals and Fractions** is a procedural task that involves dividing fractions.

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

In later grades, task **7:13 Wire Circle** is a word problem that could be solved by creating a constraint equation of the form $px + q = r$, where p, q, r, and x are all non-whole numbers.

5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

5:1

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

5:5

5:5 Write the requested values.

$4087 \times 53 = ?$	$\frac{1}{10} \div 10 = ?$	$0.4 \times 0.9 = ?$
$246 \times 914 = ?$	$\frac{7}{8} \times \frac{5}{3} = ?$	$0.75 \div 0.01 = ?$
$9744 \div 12 = ?$	$8 \times ? = 73$	$0.63 \div 0.3 = ?$
$1461 \div 6 = ?$	$3 \div \frac{1}{8} = ?$	$0.86 + 0.4 = ?$
$4 - (8 - 4) = ?$	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	$0.72 - 0.17 = ?$
	$\frac{1}{3} \div (6 \times 5) = ?$	$0.02 + 0.2 = ?$
		$0.8 - 0.55 = ?$
		$637 - 1.31 = ?$

In earlier grades, task **5:13 Frozen Yogurt Machine** is a multi-step word problem involving multiplication and division with unit fractions. Task **5:1 Juice Box Mixup** involves an unknown factor that is a fractional quotient of whole numbers. Task **5:5 Calculating** is a procedural task that includes quotients involving whole numbers and unit fractions.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

6:1 Charging Cord

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:2 Prizes, Prices, and Percents

Teacher Notes

Math Milestones™

Central math concepts

One percent of a quantity means $\frac{1}{100}$ of the quantity. And because p% of a quantity is p times as much as 1% of a quantity, it follows that p% of a quantity is $\frac{p}{100}$ of the quantity.

Unit fractions with denominators 2, 4, 5, 10, 25, and 50 have convenient equivalent fractions with denominators of 100:

$\frac{1}{2} = \frac{50}{100}$ $\frac{1}{4} = \frac{25}{100}$ $\frac{1}{5} = \frac{20}{100}$ $\frac{1}{10} = \frac{10}{100}$ $\frac{1}{25} = \frac{4}{100}$ $\frac{1}{50} = \frac{2}{100}$

These fraction equivalences imply useful fraction-percent equivalents:

$$\begin{aligned} \tfrac{1}{2} \text{ of } &= 50\% \text{ of} \\ \tfrac{1}{4} \text{ of } &= 25\% \text{ of} \\ \tfrac{1}{5} \text{ of } &= 20\% \text{ of} \\ \tfrac{1}{10} \text{ of } &= 10\% \text{ of} \\ \tfrac{1}{25} \text{ of } &= 4\% \text{ of} \\ \tfrac{1}{50} \text{ of } &= 2\% \text{ of} \end{aligned}$$

From these unit fraction equivalents, other equivalents can be found by multiplying. For example, knowing that $\frac{1}{5}$ of a quantity is 20% of the quantity, one also knows that $\frac{4}{5}$ of a quantity is 80% of the quantity.

The correspondence between decimals and percents arises by writing fractions with denominators of 100 in decimal notation; for example, $\frac{1}{100} = 0.01$, so that 1% of a quantity is 0.01 times the quantity. To say it another way, 1% of a quantity means 0.01 of each unit of the quantity.

Writing percents as decimals allows base-ten algorithms to be used for calculations, as when we calculate 75% of a $50 prize by multiplying 0.75 × 50 = 37.5 (see part (1) of task 6:2). But even without doing any base-ten calculations, it could be argued that 75% of a $50 prize is preferable to 33% of a $100 prize, because although the prize is half as small, the percent is more than twice as large. Generalizing this analysis, if we are choosing between a% of a small prize, $$S$, and b% of a large prize, $$L$, then the condition for being indifferent to the choice is when $\frac{a}{100} \times S = \frac{b}{100} \times L$. The equation for the condition of being indifferent to the choice could be written equivalently in different forms, such as $\frac{a}{b} = \frac{L}{S}$, in which form the equation states the condition that the ratio of the percents equals the inverse ratio of the prizes.

6:2 **(1)** Would you prefer 33% of a $100 prize or 75% of a $50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a $16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a $100 item adds ___ dollars to the cost. A 3% tax on a $1 item adds ___ dollars to the cost.

Answer

(1) 75% of a $50 prize. **(2)** 32. **(3)** 7%. **(4)** 0.0625, 1/16. **(5)** $17. **(6)** 3, 0.03.

Click here for a student-facing version of the task.

Refer to the Standards

6.RP.A.3c; MP.2, MP.4, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Additional notes on the design of the task

- Parts (4) and (5) are connected through the value 0.0625 = 1/16 and the price of $16.
- The second sentence of part (6) is designed to connect to the idea that 1% of a quantity means 0.01 for each 1 unit of the quantity. (So, 3% of a quantity means 0.03 for each 1 unit of the quantity, the units here being dollars.)

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: mental calculation; using ideas of times as much; using fractions and fraction equivalence; using decimals; and using fraction-decimal equivalents.

Extending the task

How might students drive the conversation further?

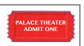
- Students could find the percent of a $50 prize that is worth the same as 33% of a $100 prize.
- By using a calculator to try combinations, students could create versions of part (1) that are "close calls," like 53% of a $43 prize vs. 24% of a $93 prize.

Related Math Milestones tasks

6:10

6:10 In the month of February 2021, there were 20 weekdays and 8 weekend days. Here are some questions about that month. **(1)** (Circle all of the correct answers.) The ratio of weekdays to weekend days was 20:8 10:4 5:2 5:7. **(2)** There were ___ times as many weekdays as weekend days. **(3)** True or false: $\frac{5}{7}$ of the days that month were weekdays. **(4)** Approximately what percent of the days that month were weekdays?

7:11



7:11 Nechama is shopping online for a ticket to a play. Website A offers a discount of $7.50 off the theater price. Website B offers a discount of 25% off the theater price. **(1)** Is it mathematically possible that Website A is a better deal than Website B? **(2)** Is it mathematically possible that Website B is a better deal than Website A? *Prove your answers.*

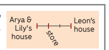
7:1

7:1 The cost of a phone is the phone's price, $264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

7:4

7:4 *"Foul Play."* The Hawks were leading the Pistons in basketball by a score of 100-98. Just as time was running out, a Pistons player tried a 3-point shot. His defender had two choices: allow the shot, or stop it by fouling the Pistons player. Fouling would give the Pistons player 3 one-point free throws. The defender chose to foul and later wondered if it was a good choice. **(1)** To analyze the defender's choice, let's assume that for the Pistons player, every 3-point shot has probability $\frac{1}{3}$ of going in, and every free throw has probability 90% of going in. **(a)** If the defender allows the shot, what is the probability that the shot wins the game as time runs out? **(b)** If the defender stops the shot by fouling, estimate the probability that the free throws win the game. **(2)** Write a paragraph arguing for or against the defender's choice, based on probability calculations and/or simulations.

5:6



5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

Task **6:10 Weekdays and Weekend Days** involves ratios, fractions, and percent.

In later grades, task **7:11 Ticket Offers** compares a percent discount to a discount in absolute dollars. Task **7:1 Phone Cost** uses percent in relation to the distributive property and algebraic expressions. Task **7:4 Foul Play** involves a probability given as a percent.

In earlier grades, tasks **5:6 Corner Store** and **5:13 Frozen Yogurt Machine** involve the extension of multiplication and division from whole numbers to fractions, with attendant ideas of scaling.

Curriculum connection

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2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:2 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

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Solution Paths

- What solution paths might you expect to see?
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Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
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Identity, Agency, and Belonging

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- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:3 South Pole Temperatures

Teacher Notes

Central math concepts

How large a number system do we need for mathematics? Students in the elementary grades have solved so wide an array of problems using only the positive fractions that it may seem unnecessary to extend the number system any further. But there are many contexts in which signed numbers are a good model for real-world quantities. Examples include temperature above/below zero, elevation above/below sea level, credits/debits, in-migration/out-migration across a state line or national border, changes in an index such as the stock market or in a rate such as the poverty rate, physical entities like positive/negative electric charge, and the study of motion itself, which is based on measuring an object's position left/right, up/down, and forward/back relative to a reference point in space.

Temperature is one important use of positive and negative numbers in everyday life. Worldwide, most people refer to temperatures as measured on the Celsius scale, but the Fahrenheit temperature scale is predominant in the United States. Temperatures on the Fahrenheit scale can be positive, negative, or 0, although the 0 point of the Fahrenheit scale does not have scientific significance. (On the Celsius scale, the 0 point indicates the temperature at which a quantity of H_2O can exist as either water or ice. For positive Celsius temperatures, H_2O is water, while for negative Celsius temperatures, H_2O is ice.)

The introduction of negative numbers extends the number line by reflecting the positive numbers across 0. Furthermore, with the introduction of negative numbers, points can be plotted in every quadrant of the entire coordinate plane relative to the origin (0, 0).

Students' statistical work in grade 6 centers on univariate measurement data. A set of univariate measurement data is a list of numbers that are measurements in context relative to a specified unit. (For a task about a set of univariate measurement data, see **6:7 Song Length Distribution**.) By contrast, the data table in task 6:3 presents a set of bivariate measurement data. A set of bivariate measurement data is conceptually a list of pairs of coordinated measurements. For example, the pair of numbers (-4, -42) records the fact that 4 hours before midnight, the temperature was -42°F. When plotted on the marvelous representational device of the coordinate plane, that single fact becomes a single point; and the totality of recorded facts forms a spatial pattern.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: choosing scales on graph paper to plot points; relating order properties of negative numbers to ideas of increase/decrease; and describing patterns.

6:3 The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

Time	Hours after Midnight	Temp °F
8:00 pm	−4	−42
9:00 pm	−3	−42
10:00 pm	−2	−41
11:00 pm	−1	−40
Midnight	0	−39
1:00 am	1	−39
2:00 am	2	−38

Plot the data on graph paper and label the plot. Describe any patterns you see.

Answer

See example plot. Characteristics of a strong data plot:

- There is an overall title (such as "South Pole Temperatures" or similar).
- There is a horizontal axis title (such as "Hours after midnight" or similar) and a vertical axis title (such as "Temperature °F" or similar).
- Axis titles indicate the units of measure for each scale.
- On both horizontal and vertical axes, numbers are present to mark the scales.
- Showing the clock times is optional.
- Connecting the data points with straight segments is optional.†
- Data values should be increasing upwards and to the right.

In describing patterns, answers may vary but should include the observation that temperatures showed a warming trend over the time period shown. Other observations that could be made are that the temperature was very cold over the time period shown, that the temperature was between -42°F and -38°F over the time period shown, and that the temperature was never observed to decrease.

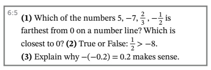
Extending the task

How might students drive the conversation further?

- Students could use rate language to describe the temperature trend during the period between 9pm and midnight.
- Students could use the internet to check current weather conditions at the South Pole and collect data over time to display and summarize.

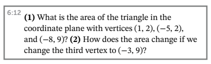
Related Math Milestones tasks

6:5



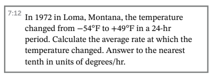
6:5 **(1)** Which of the numbers 5, −7, $\frac{2}{3}$, $-\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$. **(3)** Explain why $-(-0.2) = 0.2$ makes sense.

6:12



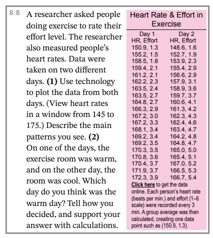
6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

7:12



7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

8:8



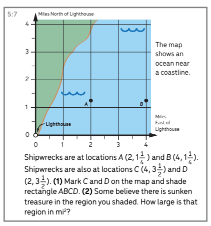
8:8 A researcher asked people doing exercise to rate their effort level. The researcher also measured people's heart rates. Data were taken on two different days. **(1)** Use technology to plot the data from both days. (View heart rates in a window from 145 to 175.) Describe the main patterns you see. **(2)** On one of the days, the exercise room was warm, and on the other day, the room was cool. Which day do you think was the warm day? Tell how you decided, and support your answer with calculations.

Heart Rate & Effort in Exercise

Day 1 HR, Effort	Day 2 HR, Effort
150.9, 1.3	148.6, 1.6
155.2, 1.5	152.7, 1.9
158.5, 1.8	153.9, 2.3
159.4, 2.1	155.4, 2.9
161.2, 2.1	156.6, 2.9
162.2, 2.3	157.9, 3.1
163.5, 2.4	158.9, 3.6
163.5, 2.7	159.7, 3.7
164.8, 2.7	160.6, 4.1
166.3, 2.9	161.3, 4.2
167.2, 3.0	162.3, 4.3
167.2, 3.3	162.4, 4.6
168.1, 3.4	163.4, 4.7
169.2, 3.4	164.2, 4.8
169.2, 3.5	164.8, 4.7
170.3, 3.5	165.0, 5.0
170.8, 3.6	165.4, 5.1
170.4, 3.7	167.0, 5.2
171.9, 3.7	166.5, 5.3
172.3, 3.9	166.7, 5.4

Click here to get the data online. Each person's heart rate (beats per min.) and effort (1–6 scale) were recorded every 3 min. A group average was then calculated, creating one data point such as (150.9, 1.3).

5:7



Shipwrecks are at locations A $(2, 1\frac{1}{4})$ and B $(4, 1\frac{1}{4})$. Shipwrecks are also at locations C $(4, 3\frac{1}{2})$ and D $(2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle $ABCD$. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi²?

Task **6:5 Positive and Negative Numbers** involves concepts of rational number ordering and magnitude. In task **6:12 Coordinate Triangle**, some of the coordinates of the triangle vertices are negative.

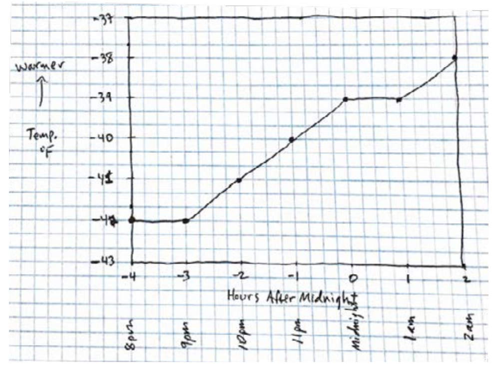
In later grades, the given information for task **7:12 Temperature Change** could be plotted in a similar way to the temperature data in task 6:3. Task **8:8 Heart Rate and Effort in Exercise** involves two distributions of bivariate data.

In earlier grades, task **5:7 Shipwrecks** involves the positive quadrant of the coordinate plane.

† From the Progression document, p. 12: "It is traditional to connect ordered pairs with line segments in such a graph, not in order to make any claims about the actual temperature value at unmeasured times, but simply to aid the eye in perceiving trends."

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Answer (continued)



Click here for a student-facing version of the task.

Refer to the Standards

6.NS.C.7, 8; MP.2. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

The data source for the table was a website giving historical data about temperatures at the Amundsen-Scott South Pole Station.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:3? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:3? In what specific ways do they differ from 6:3?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:3 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:4 Gas Mileage

Teacher Notes

Central math concepts

This task is a deep dive into a situation in which two quantities are in a proportional relationship with one another. In fact, there are two proportional relationships in the situation:

1. The **number of miles** the narrator can drive is proportional to the **number of gallons** available. In other words, there is a fixed number *e* such that the number of miles driven is *e* times the number of gallons used. The number *e* is the unit rate for this relationship.
2. The **number of gallons** the narrator will use is proportional to the **number of miles** driven. In other words, there is some fixed number *r* such that the number of gallons used is *r* times the number of miles driven. The number *r* is the unit rate for this relationship.

The different subparts of the task concern a range of concepts involved in proportional relationships, including (a) using multiplicative scaling to find an unknown quantity in a situation with a proportional relationship; (b) calculating unit rates; (c) using unit rates to find an unknown quantity in a situation with a proportional relationship; and (d) graphing a proportional relationship. The graph serves as a single picture of all the possible covarying values in the relationship.

For part (1), the values have been chosen so as to facilitate mental calculation using scaling ideas as an approach to finding an unknown quantity in a proportional relationship. For example, 57 is one-tenth of 570, and if we are driving one-tenth as far, we will use one-tenth as much gas. Meanwhile, parts (2) and (3) are designed to promote the approach of multiplying by the unit rate as an approach to finding an unknown quantity in a proportional relationship. For example, with 11 gal of gas, the number of miles we can drive can be found by using the unit rate of 38 miles per gallon: 38·11 = 418. In general then, the task is designed to promote the use of scaling and unit rates over the approach of setting up a proportion and cross-multiplying.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task:

- Mental calculation.
- Using units as guideposts to thinking about a problem.
- Thinking about measurement scales on the axes of a coordinate plane.
- Using ideas of scaling and times-as-much to multiply.

Extending the task

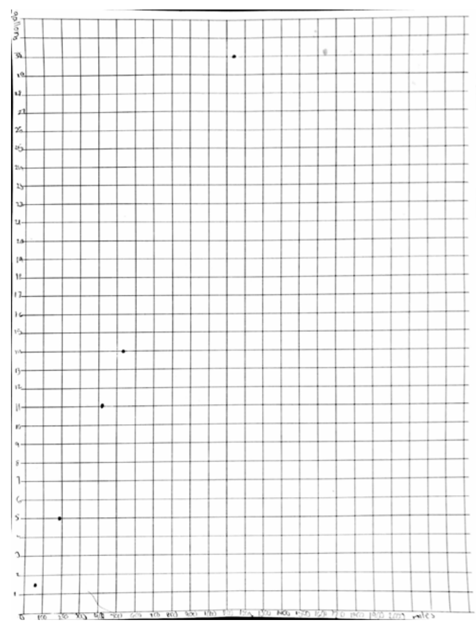
How might students drive the conversation further?

6:4

My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Answer

(1) **(a)** 1.5. **(b)** 150. **(c)** 190. **(d)** 1,140. **(2)** 38 miles per gallon and 0.026 gallon per mile. For the second unit rate, rounded values such as 0.03 gallon per mile are also correct, and the fraction form 1/38 gallon per mile is also correct. **(3)** **(a)** 14. **(b)** 418. **(4)** See example table and example graph.



Note: The axes could be interchanged and the graph would still be correct.

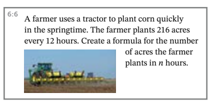
Miles driven	Gallons of gas used
57	1.5
190	5
1,140	30
532	14
418	11

Note: The left and right columns could be interchanged and the table would

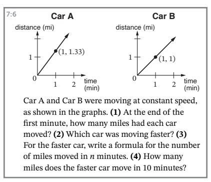
- If students did not answer part (3) by multiplying by the appropriate unit rate found in part (2), they could try that approach and verify that it leads to the same answers they obtained by other means.
- By attending to precision in creating the graph, students may notice that their plotted points appear to fall on a straight line. Students could conjecture that this will be the case for any graph of a proportional relationship. Instead of immediately receiving a definitive answer to their question, students could be told that this important question will become a focus of their work on proportional relationships in grades 7 and 8.
- For each pair of values in their table, students could calculate both possible quotients. Every row should yield the same two quotients, which are the two unit rates. Students might notice that the two constants are reciprocals of one another, and a mathematical discussion could aim for an explanation of why that is so.

Related Math Milestones tasks

6:6

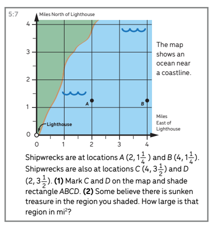

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

7:6



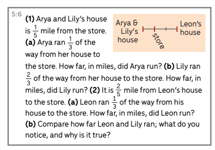
7:6 Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

5:7



5:7 Shipwrecks are at locations A $(2, 1\frac{1}{4})$ and B $(4, 1\frac{1}{4})$. Shipwrecks are also at locations C $(4, 3\frac{1}{2})$ and D $(2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle $ABCD$. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

5:6



5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

Another task for Grade 6 that prominently features a proportional relationship is **6:6 Planting Corn**.

In later grades, task **7:6 Car A and Car B** prominently features a proportional relationship.

In earlier grades, task **5:7 Shipwrecks** features the coordinate plane and Math Milestones task **5:6 Corner Store** features the multiplicative scaling idea of times-as-much.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

still be correct. Also, any order of the rows is correct.

Click here for a student-facing version of the task.

Refer to the Standards

6.RP.A.2, 3; MP.1, MP.2, MP.4, MP.5, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- The task does not provide a pre-labeled coordinate grid, because thinking about what the labels ought to be is part of the work identifying the quantities in the situation that are in a proportional relationship.
- Students are not expected to know that the points they plot should fall along a straight line; the task is designed to make this observation a potential natural outgrowth of working on the task.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:4? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 6:4? In what specific ways do they differ?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:4 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

6:4 Gas Mileage

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:5 Positive and Negative Numbers

Teacher Notes

Math Milestones™

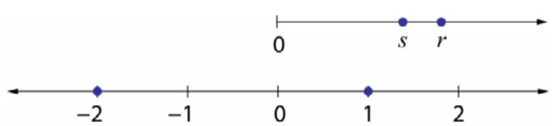
Central math concepts

This task is about disentangling notions of order from notions of magnitude. A rational number has a location on the number line, and it also has a magnitude or absolute value, which is the number's distance from 0 on the number line. For rational numbers r and s, the inequality $r > s$ is a statement of order: $r > s$ means that r is to the right of s on the number line. For example, 4 > 2 and 0 > -1.

Why is it necessary to disentangle notions of order from notions of magnitude? Consider the following two statements about two numbers r and s on the number line:

r is to the right of s.

r is farther from 0 than s.



If we restrict r and s to the nonnegative numbers, then these two statements would be equivalent (see figure, top). But for rational numbers, the two statements are not equivalent. For example, 1 is to the right of -2, but -2 is farther from 0 than 1 (see figure, bottom).

In elementary grades, the symbol - has long been used to indicate subtraction, as in the problem 7 - 2 = 5. The rational numbers introduce an additional use for this symbol, as in the example of the number -8. Here, the - sign is not indicating an operation on two numbers; rather, -8 refers to the additive inverse of 8 (the number that makes 0 when added to 8). In grade 6, prior to learning how to operate with signed numbers, the number -8 can be understood in various contexts as "the opposite of 8" (for example, if the number 8 in context represents a location 8 feet above sea level, then the number -8 represents a location 8 feet below sea level). The number -8 can be understood on the number line as the mirror reflection of the number 8 across the 0 point. Reflecting then reflecting again leaves a number unchanged, just as the opposite of the opposite of "up" is "up." In the context of grade 7 operations with rational numbers, the number -8 is fundamentally the number that makes 0 when added to 8, also the result of 0 - 8 and the result of (-1)·(8).

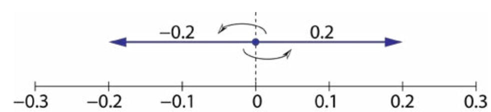
Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: number sense of fraction size; comparing fractions; locating points on a number line; and basing mathematical explanations on a number line diagram.

6:5 **(1)** Which of the numbers $5, -7, \frac{2}{3}, -\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$. **(3)** Explain why $-(-0.2) = 0.2$ makes sense.

Answer

(1) -7 is farthest from 0, and $-\frac{1}{2}$ is closest to 0. **(2)** True. **(3)** Answers may vary, may be written or spoken, and may be supported by such things as diagrams or evocations of contexts. For example, "The opposite of the opposite of a number is just the number," or "Reflecting twice takes you back to where you started." For an example of a diagram, see the figure, in which 0.2 and -0.2 are represented by arrows based at 0, with length 0.2, and direction indicated by the sign of the number.



Starting with 0.2, -0.2 is the reflection of that, and -(-0.2) is the reflection of that. Which is 0.2. 'Reflecting twice takes you back to where you started.'

Click here for a student-facing version of the task.

Refer to the Standards

6.NS.C.6, 7; MP.3, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Extending the task

How might students drive the conversation further?

- Students could create contexts that help make sense of parts (1) and (2) of the task. For example, if A has \$5 and B owes \$7, then does B owe more than A has? Is $\frac{1}{2}$ degree Fahrenheit above zero a warmer temperature than 8 degrees Fahrenheit below zero?
- Students could generalize the reasoning of part (3) to pose puzzle problems such as -(-(-8))) = ?. What if there were more than three minus signs?

Related Math Milestones tasks

6:3

6:3 The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

Time	Hours after Midnight	Temp °F
8:00 pm	−4	−42
9:00 pm	−3	−42
10:00 pm	−2	−41
11:00 pm	−1	−40
Midnight	0	−39
1:00 am	1	−39
2:00 am	2	−38

Plot the data on graph paper and label the plot. Describe any patterns you see.

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1-1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9+9)$.

(2) Show calculation 1(a) on a number line.

7:12

7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

7:5

7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

5:10

5:10 **(1)** Solve: $\frac{1}{3} = 0.1 + ?$

(2) Is there a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$? If you think so, find such a number. If you think there is no such number, explain why.

(3) Show one of the above problems and its solution on a number line.

Task **6:3 South Pole Temperatures** involves a set of time series data that uses negative values to represent times before midnight.

In later grades, task **7:9 Calculating with Rational Numbers** involves the four operations on rational numbers, and task **7:12 Temperature Change** involves arithmetic with signed rational numbers in context. Task **7:5 Is There a Solution? (Addition)** focuses on an equation that has no solution in the positive numbers but that can be solved in the rational number system.

In earlier grades, task **5:10 Number System, Number Line** explores the structure of the positive number line.

Additional notes on the design of the task

- The varied numbers in the task seek to portray positive and negative fractions and decimals as an integrated system of numbers.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:5? Locate 2–3 similar tasks in that unit. How are the tasks similar to each other, and to 6:5? In what specific ways do they differ from 6:5?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:5 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
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Language

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Identity, Agency, and Belonging

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- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:6 Planting Corn

Teacher Notes

Central math concepts

A farmer plants 216 acres every 12 hours. How many acres does the farmer plant in 6 hours?

In 2 hours?

In 3 hours?

In 18 hours?

We could solve a succession of such problems individually, and indeed part (1) of task **6:4 Gas Mileage** includes several problems along these lines—but the more such problems we solve, the more we may begin to suspect that we're repeating ourselves. Isn't there some single, essential fact about this farmer's situation that we should try to put our finger on?

The key to the farmer's situation is to think about what happens in 1 hour. In 1 hour, the farmer plants 216 ÷ 12 = 18 acres. 18 acres per hour is the *unit rate* for the proportional relationship between the number of acres and the number of hours. Why is the unit rate helpful? To see why, first set aside the unit rate and consider again that problem of how many acres the farmer plants in 6 hours. A solution might be based on the fact that because 6 hours is half of 12 hours, the farmer will plant half of 216 acres in 6 hours: in other words, 108 acres. In finding that solution, we have used the given information (216 acres, 12 hours) together with a scaling argument: in half the time, the farmer finishes half the work. But what if the given information had been more useful? What if, instead of specifying that the farmer plants 216 acres every 12 hours, the problem had said, "A farmer plants 18 acres every 1 hour." Then the problem of how many acres are planted in 6 hours would certainly be no harder: in 6 times the time, the farmer finishes 6 times the work, or 6 × 18 = 108 acres. That's the same answer we obtained before.

The advantage of the unit rate becomes clearer if we consider a question like, "How many acres are planted in 5 hours?" Using the originally given information, we could certainly say that in $\frac{5}{12}$ of the time, $\frac{5}{12}$ of the work gets done: $\frac{5}{12} \times 216$ acres are planted. Equally well, we could set up a proportion and solve it: $\frac{5}{12} = \frac{A}{216}$. However, the better information that "a farmer plants 18 acres every 1 hour" is so helpful, we might as well just use that information directly and argue that in 5 times as many hours, 5 times as much work gets done: therefore 5 × 18 = 90 acres are planted. The unit rate is essentially an improved version of the originally given information. The problem didn't originally provide the unit rate to us, but we are still entitled to provide it to ourselves.

Knowing that 18 acres are planted every hour, we can find the number of acres planted in any number of hours, *n*. That's because, in *n* times as many hours, *n* times as much work gets done:

Number of acres planted in *n* hours = *n* times the number of acres planted in 1 hour

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in *n* hours.

Answer

$A = 18n$ (students might use a different letter for the acreage variable or use different conventions to indicate multiplication such as $18 \times n$ or $18 \cdot n$ or $18 * n$).

Click here for a student-facing version of the task.

Refer to the Standards

6.RP.A, 6.EE.C.9; MP.2, MP.4, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- The numbers 216 and 12 were chosen because of the many divisibilities they offer. This not only makes the unit rate a whole number, but it also creates the possibility of students creating an informative table using only whole-number values.
- A farmer in this newspaper article planted 312 acres in a 16-hour day, which is a rate of 19.5 acres per hour.

Number of acres planted in n hours = n × unit rate

Number of acres planted in n hours = n × 18.

To reach this conclusion, we didn't have to "set up a proportion." But for the sake of seeing the problem from many sides, let's now approach the problem that way. In doing so, we can use the variables A and n to stand for the number of acres and the number of hours. Then the proportion is the equation,

$$\frac{216}{12} = \frac{A}{n}.$$

Instead of cross-multiplying right away, let's first complete the division on the left-hand side:

$$18 = \frac{A}{n}.$$

If we *now* cross-multiply, then we obtain the answer to the task: $18n = A$.

The equation $18 = \frac{A}{n}$ is a more informative statement about the situation than the proportion $\frac{216}{12} = \frac{A}{n}$, because $18 = \frac{A}{n}$ shows the value of the unit rate.

Setting up and solving a proportion only ever gives a single number as the result. But task 6:6 is about the functional thinking inherent in a proportional relationship. That's why the answer to the problem is a formula, not a single number.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: mental calculation; using ideas of scaling and times-as-much; and defining and using a variable.

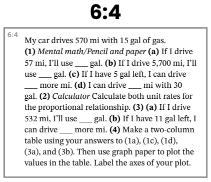
Extending the task

How might students drive the conversation further?

- Students can use the equation $A = 18n$ to determine how many hours or days it would take to plant 1,000 or 10,000 acres of corn.
- How many hours would be spent planting 90 million acres of corn, which is about how many acres are planted annually in the United States?
- If a farmer works a 12-hour day during planting season, then how many days would that much planting take? If planting season lasts only about a month, then how many farmers are needed to plant all that corn?

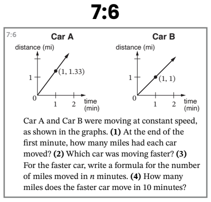
Related Math Milestones tasks

6:4



6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

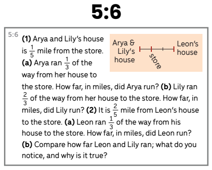
7:6



7:6

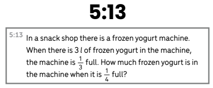
Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

5:6



5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

5:13



5:13 In a snack shop there is a frozen yogurt machine. When there is 3 l of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

Another Math Milestones task for grade 6 that prominently features a proportional relationship is **6:4 Gas Mileage**. Whereas in task 6:6 a function

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:6? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 6:6? In what specific ways do they differ from 6:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

equation expresses the relationship, in task 6:4 a table and a graph are used to analyze the relationship.

In later grades, task **7:6 Car A and Car B** prominently features a proportional relationship and the idea of unit rate, with a graphical representation as central.

In earlier grades, tasks **5:6 Corner Store** and **5:13 Frozen Yogurt Machine** situate the multiplicative scaling idea of times-as-much in context.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:7 Song Length Distribution

Teacher Notes

Central math concepts

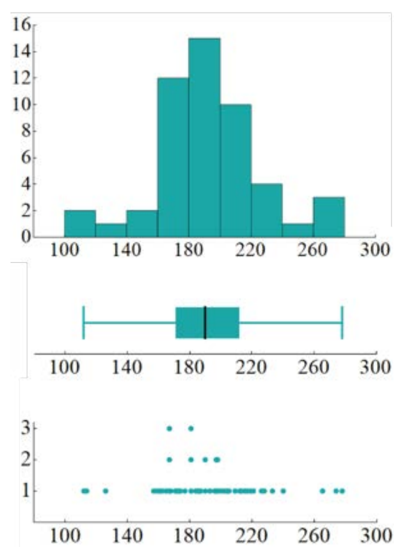
As explained in the relevant *Progression document*, statistical reasoning is a four-step investigative process:

- Formulate questions that can be answered with data
- Design and use a plan to collect relevant data
- Analyze the data with appropriate methods
- Interpret results and draw valid conclusions from the data that relate to the questions posed.†

In contrast to mathematical questions about mathematical objects, *statistical questions* anticipate variability in the data related to the question and account for it in the answers. CCSS 6.SP.A.1 gives the following example:

> *"How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

A set of data collected to answer a statistical question has a *distribution* that can be described by its center, spread, and overall shape. Ways of picturing distributions include creating histograms, box plots and dot plots. (See the figure for an example of each kind of display using the song length data.)

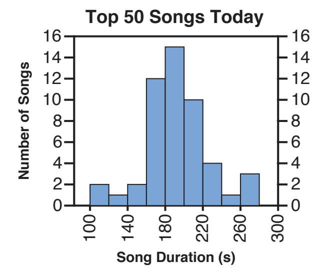


All three types of plot have a measurement scale along the horizontal axis. In a dot plot or histogram, the vertical scale is a count scale. In a box plot, the vertical scale is not numerically meaningful.

Students used dot plots in the elementary grades. In a dot plot, which is useful primarily for smaller data sets, every data point is shown. In a histogram, data values are grouped into bins and counted. The bins are not given and must be imposed on the data, usually by technology based on an algorithm. In a box plot, the data values are not shown; but by indicating the extreme values, the quartiles, and the median, the box plot visually indicates the width and clustering of the distribution of data values.

Box plots and histograms are sophisticated representations of data. In either case, reading a single fact from the display involves thoroughly comprehending the data, the context, and the conventions of the representation. For example, the tallest rectangle in the histogram represents the fact that 15 songs in the data set had durations of at least 180 seconds but less than 200 seconds.

6:7 **(1)** Look up the 50 top songs on a music streaming service. Type each song's duration into a spreadsheet. **(2)** Write a sentence about the data giving a measure of center and a measure of variability. **(3)** Make a histogram of the data.* **(4)** Write a sentence describing the overall pattern of the distribution and any striking deviations from the overall pattern. **(5)** Imagine that one year from now, you go back online and repeat (1)–(4). In what ways would you expect the data distribution to look similar? What differences would you expect to see?



*Use this histogram for (4) and (5) if you don't do (3).

Answer

(1) Data sets may vary; see example. **(2)** Answers may vary. Examples: "The mean song duration was 192 seconds, with a mean absolute deviation of 25 seconds"; "The median song duration was 190 seconds, with an interquartile range of 40 seconds." **(3)** Answers may vary; for an example, see the histogram provided in the task. **(4)** Answers may vary but should include the observation that song durations cluster near the center. **(5)** Answers may vary but could include hypotheses about whether the mean/median song duration would be similar or different to today's, and whether the variability in top song durations would be similar or different to today's. An opinion could be expressed about whether extreme song length (very short or very long) is a disadvantage for popularity. Answers should not suggest that the data will look precisely the same one year from now.

Click here for a student-facing version of the task.

A measure of center for a data set summarizes all of its values with a single number, and a measure of variability describes how its values vary with a single number (CCSS 6.SP.A.3). Inherent in the notion of a *summary* is loss of information; for example, if we say that "The mean song duration was 192 seconds, with a mean absolute deviation of 25 seconds," then we have left out an enormous number of details about the data set, but we have also said something useful about the data set. Measures of center and measures of variability thus allow us to refer to complicated realities in simplified terms. In that sense, these measures are important mathematical models.

In practical terms, measures of center and measures of variability are most useful when the data set in question has a large number of values. If there are only a few data values, then no summary is needed. Authentic statistics education should therefore include some work with relatively large data sets, including using technology to record, organize, analyze, and display those data sets.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: thinking about data in context; calculating measures of center and measures of variation; describing patterns in distributions of univariate measurement data; and using technology.

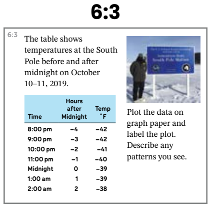
Extending the task

How might students drive the conversation further?

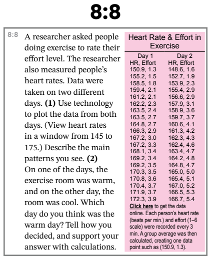
- Students could discuss the relative effectiveness of a dot plot, box plot, and histogram for displaying the distribution in ways that promote understanding of the patterns.
- Students could compare the distributions from two different genres to see if the distributions are appreciably different (CCSS 7.SP.B.3).

Related Math Milestones tasks

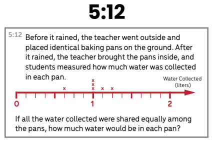
6:3



8:8



5:12



Task **6:3 South Pole Temperatures** involves a distribution of bivariate measurement data.

In later grades, task **8:8 Heart Rate and Effort in Exercise** involves two distributions of bivariate data.

In earlier grades, task **5:12 Rain Measurements** involves a set of univariate measurement data with a calculation that prefigures the mean as a measure of center.

Refer to the Standards

6.SP; MP.3, MP.4, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency, Application

Additional notes on the design of the task

- Task 6:7 uses a selection criterion of the nationwide top 50 songs, but an alternative could be for students to choose a more specific music genre/chart they know about or enjoy.
- The data source for the histogram in task 6:7 was a music streaming service (accessed in 2019).

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:7? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:7? In what specific ways do they differ from 6:7?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:7 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core State Standards Writing Team (2011), *Progressions for the Common Core State Standards in Mathematics (draft) 6–8 Statistics and Probability.*

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6:7 Song Length Distribution

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

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Solution Paths

- What solution paths might you expect to see?
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Identity, Agency, and Belonging

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6:8 Evaluating an Expression

Teacher Notes

Central math concepts

Working with expressions involves looking for and making use of structure. Just as the expression 4 × 76 × 25 rewards pausing before diving in and multiplying the factors in given order from left to right, the expression in task 6:8 rewards pausing before diving in and substituting into the given expression. And just as rewriting 4 × 76 × 25 in the equivalent form 4 × 25 × 76 serves a purpose of making a calculation easier, combining like terms in task 6:8 serves the same purpose. Finally, just as rewriting 4 × 76 × 25 as 4 × 25 × 76 relies on properties of operations (associativity and commutativity of multiplication), collecting like terms in task 6:8 also relies on properties of operations, in particular the distributive property:

$$
\begin{aligned}
& 0.96r + 0.04r - r \\
&= 0.96r + 0.04r - 1r \\
&= (0.96 + 0.04 - 1)r \\
&= (1 - 1)r \\
&= 0.
\end{aligned}
$$

This shows that the expression will evaluate to 0 no matter what value of *r* is substituted into it.

There isn't a standard algorithm for evaluating or transforming algebraic expressions; instead, there are choices to make. Those choices require comprehension of the structure of expressions, as well as linguistic fluency with the syntax of expressions and their conventions (such as omitting the multiplication sign, using the same symbol for subtraction as for negation, or understanding that a term "*r*" has a coefficient of 1). As examples like 4 × 76 × 25 or 4,999 + 12 illustrate, calculation in the elementary grades was never only algorithmic,[†] and in the middle grades and high school it seldom ever is.[‡]

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: adding two-digit decimals; interpreting written conventions of algebraic expressions; using the distributive property; and viewing expressions as objects with structure.

Extending the task

How might students drive the conversation further?

- Students could see what happens if 1 is substituted for *r* in the given expression. (Is substituting 1 a kind of "trick" for collecting like terms?)
- Suppose the second coefficient in the given expression is changed from 0.04 to 0.05. What will be the result of substituting $r = 11{,}000$?

6:8	*Pencils down* If $r = 1.748$, what is the value of $0.96r + 0.04r - r$?

Answer

0.

Click here for a student-facing version of the task.

Refer to the Standards

6.EE.A; MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency

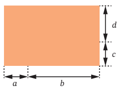
Additional notes on the design of the task

- The combination of "pencils down" together with the use of decimals to thousandths (1.748) is intended to create an incentive to look for a labor-saving approach. However, the purpose of task 6:8 isn't to differentiate between students who do or don't think of collecting like terms. Rather, the purpose is to help all students see the power of looking at the structure of an expression and seeing what opportunities it affords.
- The first two terms in the expression might make sense and/or lead to ideas if interpreted as, "96% of something plus 4% of that thing."

Related Math Milestones tasks

6:11

6:11 The diagram shows a rectangle. The variables a, b, c, and d are lengths in meters. **(1)** Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.



6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

Task **6:11 Area Expressions** involves transforming expressions into equivalent forms using properties of operations. **Task 6:13 Is There a Solution? (Multiplication)** is a task that, like 6:8, surfaces concepts in a topic that involves procedural fluency.

7:1

7:1 The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$.
(2) Show calculation 1(a) on a number line.

7:8

7:8 In 2018, an oil company rented an oil rig for \$100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for \$70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make \$30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

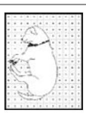
8:2

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

In later grades, tasks **7:1 Phone Cost**, **7:9 Calculating with Rational Numbers**, and **7:8 Oil Business** also involve the distributive property. Task **8:2 Pottery Factory** involves a situation that can be modeled with an equation in which properties of operations including the distributive property would be used in the process of solving for an unknown value.

3:2

3:2



The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.

Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.

12×14, 11×14, 12×15, 11×15

4:5

4:5 **(1a–f)** Write the values of the products. Compare answers with a classmate.

$4 \times \frac{1}{7} =$ ___ (a)
$6 \times \frac{4}{7} =$ ___ (b)
$86 \times \frac{1}{86} =$ ___ (c)
$6 \times \frac{8}{2} =$ ___ (d)
$9 \times \frac{1}{9} =$ ___ (e)
$9 \times \frac{2}{9} =$ ___ (f)

(1g) Which answer is twice as much as the answer for (e)?
(1h) Which answer is six times as much as the answer for (a)?
(1i) Which two answers are equal?
(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

5:14

5:14 Brandon was reading his math book. He saw the equation $\frac{3}{4} \times (4 + \frac{1}{2}) = 3 + \frac{3}{8}$. He said, "I don't get it—where did the 3 and the $\frac{3}{8}$ come from?" Write an explanation that could answer Brandon's question.

5:1

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

In earlier grades, tasks **3:2 Hidden Rug Design**, **4:5 Fraction Products and Properties**, and **5:14 Brandon's Multiplication Equation** are tasks involving expressions as objects with structure. Task **5:1 Juice Box Mixup** has an interpretation in terms of multiplication distributing over subtraction.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:8? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:8? In what specific ways do they differ from 6:8?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:8 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?[3]

† National Research Council. 2001. Adding It Up: Helping Children Learn Mathematics. Washington, DC: The National Academies Press. Page 121.

‡ William McCallum (2008), "Mindful Manipulation: What Algebra Do Students Need for Calculus?" (presentation)

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

6:8 Evaluating an Expression

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

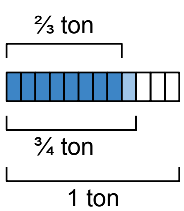
6:9 Truckload of Gravel

Teacher Notes

Central math concepts

Because $\frac{2}{3}$ is less than $\frac{3}{4}$, the answer to task 6:9 must be a fraction less than 1. An analogous problem posed with whole numbers might read, "How much of a 9-liter container is 8 liters of water?" The solution to that problem is the unknown factor in $\square \times 9 = 8$, which is the quotient $8 \div 9$. Likewise, the solution to task 6:9 is the unknown factor in $\square \times \frac{3}{4} = \frac{2}{3}$, which is the quotient $\frac{2}{3} \div \frac{3}{4}$. Students used such unknown-factor reasoning in earlier grades, for example when they thought along the lines of, "7 times something is 42, so the something must be $42 \div 7$."

Fraction equivalence and unit fractions play a role, visible or invisible, in most fraction situations. For example, suppose we replace the fractions $\frac{3}{4}$ and $\frac{2}{3}$ in task 6:9 by equivalent fractions with the same denominator, $\frac{9}{12}$ and $\frac{8}{12}$. Think of $\frac{1}{12}$ ton as a new unit of gravel, a "twelfth-ton" of gravel. Then the question is equivalent to, "How much of a 9-unit quantity is 8 units?" This may suggest the answer $\frac{8}{9}$ or the process $8 \div 9$. Thinking about twelfths of a ton may also help to create a diagram of the situation. In this diagram, $\frac{2}{3}$ ton is shown as 8 parts in a partition of $\frac{3}{4}$ ton into 9 parts.



Algebraically, the structure of task 6:9 corresponds to the equation $F \times \frac{3}{4} = \frac{2}{3}$, where F is the desired fraction. Dividing both sides of the equation by $\frac{3}{4}$ produces the unknown factor: $F = \frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$.

Calculating the quotient $\frac{2}{3} \div \frac{3}{4}$ is equivalent to calculating the product $\frac{2}{3} \times \frac{4}{3}$. This circumstance is connected to a second way to solve the equation $F \times \frac{3}{4} = \frac{2}{3}$, which is to multiply both sides of the equation by $\frac{4}{3}$:

$$\frac{4}{3} \times \left(F \times \frac{3}{4}\right) = \frac{4}{3} \times \frac{2}{3}$$
$$\frac{4}{3} \times \left(\frac{3}{4} \times F\right) = \frac{4}{3} \times \frac{2}{3}$$
$$\left(\frac{4}{3} \times \frac{3}{4}\right) \times F = \frac{4}{3} \times \frac{2}{3}$$
$$1 \times F = \frac{8}{9}$$
$$F = \frac{8}{9}.$$

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

Answer

$\frac{2}{3}$ ton of gravel is $\frac{8}{9}$ of a $\frac{3}{4}$-ton truckload of gravel.

Click here for a student-facing version of the task.

Refer to the Standards

6.NS.A.1, 6.EE.B.7; MP.2, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- Task 6:9 isn't well thought of as a proportional relationships problem. The context doesn't feature two variable quantities, just a single unknown value. The equation, $F \cdot \frac{3}{4} = \frac{2}{3}$ is a constraint equation, not a function equation. That said, the observation in Central Math Concepts about scaling up shows that fraction quotients and fraction equivalence can sometimes involve versions of proportional thinking.

This solution approach relates to the context in the following way. Imagine scaling up both of the quantities in task 6:9 by a factor of four-thirds. Because $\frac{4}{3} \times \frac{3}{4} = 1$ and $\frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$, the scaled-up problem will say:

> How much of a 1-ton truckload of gravel is $\frac{8}{9}$ ton of gravel?

The answer to this question, $\frac{8}{9}$, is plain, and this answer must also be the answer to the original question in task 6:9, because both quantities were scaled by the same factor. It is as if we have changed the units in the problem from truckloads measuring $\frac{3}{4}$ ton to more convenient units of "big-truckloads" measuring 1 ton.

For an illuminating discussion of fraction division with numerous diagrams and examples, see the 2017 series of blog posts by William McCallum and Kristin Umland on MathematicalMusings.org (part 1, part 2, part 3, part 4).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: thinking about unit fractions in context; using ideas of scaling and times-as-much; and defining a variable to create and solve a constraint equation.

Extending the task

How might students drive the conversation further?

- Students who used different approaches or who drew mathematically different diagrams to represent the problem could explain their thinking to each other.
- Students could discuss whether it makes sense that the answer, $\frac{8}{9}$, is less than 1.

Related Math Milestones tasks

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

6:2

6:2 **(1)** Would you prefer 33% of a \$100 prize or 75% of a \$50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a \$16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a \$100 item adds ___ dollars to the cost. A 3% tax on a \$1 item adds ___ dollars to the cost.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

6:14

6:14 *Pencil and paper* **(1)** $81.53 \div 3.1 = ?$ **(2)** $\frac{7}{8} \div \frac{2}{3} = ?$ **(3)** Check both of your answers by multiplying.

Like task 6:9, task **6:1 Charging Cord** is a word problem with an unknown factor that is a quotient of fractions. Task **6:13 Is There a Solution? (Multiplication)** deals with the existence and nature of solutions of a particular equation with form $ax = b$. Task **6:2 Prizes, Prices, and Percents** includes percent problems with the mathematical structure of an unknown factor problem (parts (2) and (3)). Task **6:4 Gas Mileage** includes several scaling problems in the context of a single proportional relationship. Task **6:14 Dividing Decimals and Fractions** is a procedural task that involves dividing fractions.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:9? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 6:9? In what specific ways do they differ from 6:9?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:9 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

In later grades, task **7:13 Wire Circle** is a word problem that could be solved by creating a constraint equation of the form $px + q = r$, where p, q, r, and x are all non-whole numbers.

5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

5:1

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

5:5

5:5 Write the requested values.

$4087 \times 53 = ?$	$\frac{1}{10} \div 10 = ?$	$0.4 \times 0.9 = ?$
$246 \times 914 = ?$	$\frac{7}{8} \times \frac{5}{3} = ?$	$0.75 \div 0.01 = ?$
$9744 \div 12 = ?$	$8 \times ? = 73$	$0.63 \div 0.3 = ?$
$1461 \div 6 = ?$	$3 \div \frac{1}{8} = ?$	$0.86 + 0.4 = ?$
$4 - (8 - 4) = ?$	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	$0.72 - 0.17 = ?$
	$\frac{1}{3} \div (6 \times 5) = ?$	$0.02 + 0.2 = ?$
		$0.8 - 0.55 = ?$
		$637 - 1.31 = ?$

In earlier grades, task **5:13 Frozen Yogurt Machine** is a multi-step word problem involving multiplication and division with unit fractions. Task **5:1 Juice Box Mixup** involves an unknown factor that is a fractional quotient of whole numbers. Task **5:5 Calculating** is a procedural task that includes quotients involving whole numbers and unit fractions.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

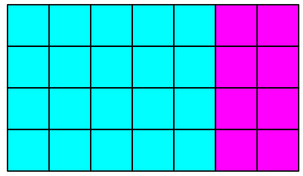
Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:10 Weekdays and Weekend Days

Teacher Notes

Central math concepts



Schematic of days that month

If we say that "The ratio of weekdays to weekend days in February 2021, was 20:8," then we haven't said much more than that there were 20 weekdays and 8 weekend days that month. The word "ratio" isn't doing much work in that sentence. On the other hand, if we say that the ratio of weekdays to weekend days in February 2021, was 5:2, then we are invoking ideas of scaling: it's not that there were 5 weekdays and 2 weekend days that month, but rather that *for every* 5 weekdays there were 2 weekend days.

The phrase "for every" in the previous sentence is reminiscent of the common multiplication phrase "for each"—as in, "For each tutor there were 2 students." Just as the total number of students is double the number of tutors, the total number of weekend days is double the number of groups of 5 weekdays in the month. Equivalently, the total number of weekend days is $\frac{2}{5}$ of the number of weekdays. Reversing that comparison, the total number of weekdays is $\frac{5}{2}$ of the number of weekend days.

In a purely technical sense, the ratio a:b isn't a number, because it's a symbol that consists of two numbers separated by a colon. But always to insist on that level of precision in language would be counterproductive, seeing as it's very common in mathematics to say such things as, "The ratio of a circle's circumference to its diameter is π" (and π is a number), or "The slope of a line in the coordinate plane is the ratio of the rise over the run" (and slope is a number). Such usages are reasonable and productive in view of the fact that ratio a:b can certainly be said to have a value, namely the number $\frac{a}{b}$ (or equivalently, the numerical result of the division $a \div b$). Two ratios that are equivalent to a:b, say $(4a)$:$(4b)$ and $(2a)$:$(2b)$, have equal values, because $\frac{4a}{4b}$ and $\frac{2a}{2b}$ are equal as numbers.

In a table of equivalent ratios, all the ratios have the same value. That connects constant ratios to their important sequel, proportional relationships. As an example in context, in a recipe for dry rub that specifies 4 tablespoons of paprika and 2 tablespoons of brown sugar, the ratio of paprika to brown sugar is constant, which guarantees a constant paprika-brown-sugar flavor profile no matter how large or small the quantity of dry rub we make. If we graphed the amount of paprika against the amount of brown sugar for different total quantities of dry rub, the graphed points would lie on a straight line whose constant ratio of rise to run has the same value as the constant ratio in the recipe.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: using ideas of times as much; using fractions; calculating percent given the part and the whole.

6:10 In the month of February 2021, there were 20 weekdays and 8 weekend days. Here are some questions about that month. **(1)** (Circle all of the correct answers.) The ratio of weekdays to weekend days was 20:8 10:4 5:2 5:7. **(2)** There were ___ times as many weekdays as weekend days. **(3)** True or false: $\frac{5}{7}$ of the days that month were weekdays. **(4)** Approximately what percent of the days that month were weekdays?

Answer

(1) 20:8, 10:4, and 5:2. **(2)** $\frac{5}{2}$ (or equivalent forms). **(3)** True.

(4) Approximately 70% (or a more precise value).

Click here for a student-facing version of the task.

Refer to the Standards

6.RP.A.1; MP.2, MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- In part (1) the ratio not circled, 5:7, isn't circled because it makes a different comparison, namely weekdays to total days. The value of that ratio is $\frac{5}{7}$, which is the fraction that appears in the true statement of part (3), "$\frac{5}{7}$ of the days that month were weekdays."

Extending the task

How might students drive the conversation further?

- Students might wonder if other months have the same ratios as February 2021. What is the ratio of weekdays to weekend days during the current month? What month(s) next year have the greatest percentage of weekend days?
- What is the percentage of weekend days for all of the next calendar year? (What would you estimate it to be?)

Related Math Milestones tasks

6:2

6:2 **(1)** Would you prefer 33% of a $100 prize or 75% of a $50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a $16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a $100 item adds ___ dollars to the cost. A 3% tax on a $1 item adds ___ dollars to the cost.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

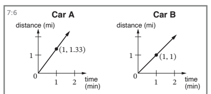
6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

Task **6:2 Prizes, Prices, and Percents** involves percent as a ratio per 100. Tasks **6:4 Gas Mileage** and **6:6 Planting Corn** involve proportional relationships.

7:6

7:6



Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does that car move in 10 minutes?

7:8

7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note, 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

7:7

7:7 If the speed limit in Canada is 100 km/hr and you are driving 65 mph, are you over or under the limit? By how much?

7:12

7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24 hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

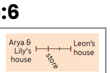
7:2

7:2 A utility pole 24 feet long has $28\frac{1}{4}$ inch circumference at the top and $47\frac{1}{8}$ inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale $\frac{1}{4}$ inch = 1 foot.

In later grades, tasks **7:6 Car A and Car B** and **7:8 Oil Business** involve proportional relationships. In task **7:7 Speed Limit**, two unit rates are compared, and in task **7:12 Temperature Change**, an average rate is calculated. In task **7:2 Utility Pole Scale Drawing**, lengths in the drawing have a constant ratio with lengths measured directly on the physical object.

5:6

5:6 **(1)** Arya and Lily's house is $\frac{1}{5}$ mile from the store. **(a)** Arya ran $\frac{1}{3}$ of the way from her house to the store. How far, in miles, did Arya run? **(b)** Lily ran $\frac{2}{3}$ of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is $\frac{2}{5}$ mile from Leon's house to the store. **(a)** Leon ran $\frac{1}{3}$ of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?



5:13

5:13 In a snack shop there is a frozen yogurt machine. When there is 3 *l* of frozen yogurt in the machine, the machine is $\frac{1}{3}$ full. How much frozen yogurt is in the machine when it is $\frac{1}{4}$ full?

In earlier grades, tasks **5:6 Corner Store** and **5:13 Frozen Yogurt Machine** involve the extension of multiplication and division from whole numbers to fractions, with attendant ideas of scaling.

Additional notes on the design of the task (continued)

- Task 6:10 was inspired by a home conversation with a sixth-grader who had opinions about what would be a fair ratio of school days to weekend days.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:10? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:10? In what specific ways do they differ from 6:10?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:10 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

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Anticipating and responding to student thinking about the task

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On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:11 Area Expressions

Teacher Notes

Central math concepts

An expression records operations with numbers and with letters standing for numbers. To restate this in more active terms, one creates an expression by calculating with variables as if they were numbers. For example, suppose three items are purchased with prices a, b, and c, and suppose there is city, county, and state tax on the purchase, with respective tax rates given by r, s, and t. Then the total tax on the purchase could be calculated as $(r + s + t)\cdot(a + b + c)$.

A conceptually distinct way to figure the total tax on the purchase would be to compute the total tax on each item and sum those amounts: $r\cdot(a + b + c) + s\cdot(a + b + c) + t\cdot(a + b + c)$. The expressions $(r + s + t)\cdot(a + b + c)$ and $r\cdot(a + b + c) + s\cdot(a + b + c) + t\cdot(a + b + c)$ are equivalent, meaning they would have the same values no matter what values of the variables are substituted into them. This is intuitively plausible in the sales tax situation, and it's mathematically guaranteed by the distributive property. For that matter, the distributive property would apply even if some or all of "tax rates" were negative (this could conceivably happen if there were a rebate program of some kind in effect).

The properties of operations can be used not only to see that two given expressions are equivalent, but also more in a more active fashion to transform one given expression into another, equivalent expression—ideally an expression that is simpler than, or that adds insight to, the given one. For example, a faster than usual way to calculate the average of 84 and 28 would be to add 42 + 14; this method works because $\frac{(a+b)}{2} = \frac{1}{2}a + \frac{1}{2}b$. As another example, if $y = mx + b$ is a linear function with $b \neq 0$, then although y is not proportional to x, changes in y are proportional to changes in x, and this can be proved by applying properties of operations: if $y_1 = mx_1 + b$ and $y_2 = rx_2 + b$ are two different y values, then the change in quantity y equals

$$\begin{aligned} & y_2 - y_1 \\ &= (mx_2 + b) - (mx_1 + b) \\ &= mx_2 - mx_1 \\ &= m(x_2 - x_1) \end{aligned}$$

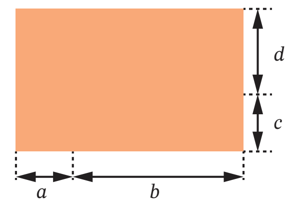
so that the change in y is always a constant multiple m of the change in x.

The transformations in task 6:11 centrally involve the distributive property, which states the mathematical relationship between multiplication and addition:

$$a(b + c) = ab + ac.$$

The distributive property is an identity, an equation that is true for all possible values of its variables. The left-hand side of the identity is a product of two terms (one of which is a sum), while the right-hand side of the identity is a sum of two terms (both of which are products). The distributive property allows us to rewrite sums as products and rewrite products as sums.

6:11 The diagram shows a rectangle. The variables a, b, c, and d are lengths in meters.



(1) Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

Answer

(1) Any three expressions equivalent to $(a + b)(c + d)$. Examples: $a(c + d) + b(c + d)$, $db + ca + ad + bc$, $(c + d)(b + a)$. The multiplication symbol × or * or · may be included or omitted. **(2)**, **(3)** Answers may vary. For example, applying the distributive property to $(a + b)(c + d)$ can result in $(a + b)c + (a + b)d$, which shows that these two expressions are equivalent. Another example: applying the distributive property to $ac + ad + bc + bd$ can result in $ac + ad + b(c + d)$, which shows that these two expressions are equivalent.

[Click here](#) for a student-facing version of the task.

Refer to the Standards

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Aspect(s) of rigor:

Concepts

Many familiar techniques in algebra can be understood as applications of the distributive property:

- Distributing. When we distribute r in the example $r(k + p + 2) = rk + r(p + 2)$, we are applying the distributive property to rewrite a product of two terms as a sum of two terms. Note that we could apply the distributive property again to rewrite $r(p + 2)$ as $rp + 2r$. That would result in the identity $r(k + p + 2) = rk + rp + 2r$.
- Factoring. When we factor out y in the example $xy + yz = y(x + z)$, we are applying the distributive property to rewrite a sum of two terms as a product of two terms.
- Collecting like terms. When we collect like terms in the example $\frac{1}{8}x + -\frac{1}{2}x + \frac{1}{4}x = (\frac{1}{8} + -\frac{1}{2} + \frac{1}{4})x$, we are applying the distributive property to rewrite a sum of three terms as a product of two terms. Of course we could continue to evaluate the sum $\frac{1}{8} + -\frac{1}{2} + \frac{1}{4} = -\frac{1}{8}$, which would result in the identity $\frac{1}{8}x + -\frac{1}{2}x + \frac{1}{4}x = -\frac{1}{8}x$.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: decomposing a rectangle into rectangles; applying area formulas and area reasoning; and viewing expressions as objects with structure.

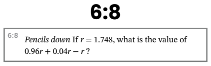
Extending the task

How might students drive the conversation further?

- Students could create two equivalent expressions for the perimeter of the rectangle and explain how one can be transformed into the other by applying properties of operations.
- If students know the triangle area formula $A = \frac{1}{2}b\,h$, and if they have used the trapezoid area formula $A = \frac{1}{2}(b_1 + b_2)$h, then they could apply the distributive property to the expression $\frac{1}{2}(b_1 + b_2)h$, and interpret the resulting expression as the sum of two triangle areas. Where are these triangles in the trapezoid? (Because a trapezoid can be decomposed into triangles, the trapezoid area formula isn't as important as the area triangle area formula.)

Related Math Milestones tasks

6:8


6:8 *Pencils down* If $r = 1.748$, what is the value of $0.96r + 0.04r - r$?

Task **6:8 Evaluating an Expression** involves an expression that could be rewritten in a more convenient form by making use of its structure.

Additional notes on the design of the task

Area in the task could connect to area models students may have used for multi-digit multiplication. The partial-products calculation $45 \times 63 = 40 \times 60 + 40 \times 3 + 5 \times 60 + 5 \times 3$ corresponds to task 6:11 with $a = 40$, $b = 5$, $c = 60$, and $d = 3$.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:11? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:11? In what specific ways do they differ from 6:11?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

7:1

7:1 The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$.

(2) Show calculation 1(a) on a number line.

7:8

7:8 In 2018, an oil company rented an oil rig for \$100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for \$70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make \$30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

8:2

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

In later grades, tasks **7:1 Phone Cost**, **7:9 Calculating with Rational Numbers**, and **7:8 Oil Business** also involve the distributive property. Task **8:2 Pottery Factory** involves a situation that can be modeled with an equation in which properties of operations including the distributive property would be used in the process of solving for an unknown value.

5:1

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

4:5

4:5 **(1a–f)** Write the values of the products. Compare answers with a classmate.

(1g) Which answer is twice as much as the answer for (e)?

(1h) Which answer is six times as much as the answer for (a)?

(1i) Which two answers are equal?

(2) Zoe was reading her math book. She saw the equation $6 \times (4 + \frac{1}{2}) = 24 + 3$. She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.

$4 \times \frac{1}{7} = \underline{\quad}$ (a)

$6 \times \frac{4}{7} = \underline{\quad}$ (b)

$86 \times \frac{1}{86} = \underline{\quad}$ (c)

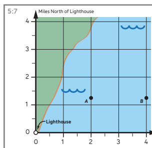
$6 \times \frac{8}{2} = \underline{\quad}$ (d)

$9 \times \frac{1}{9} = \underline{\quad}$ (e)

$9 \times \frac{2}{9} = \underline{\quad}$ (f)

5:7

5:7



The map shows an ocean near a coastline.

Shipwrecks are at locations A $(2, 1\frac{1}{4})$ and B $(4, 1\frac{1}{4})$. Shipwrecks are also at locations C $(4, 3\frac{1}{2})$ and D $(2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle $ABCD$. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

5:14

5:14 Brandon was reading his math book. He saw the equation $\frac{3}{4} \times (4 + \frac{1}{2}) = 3 + \frac{3}{8}$. He said, "I don't get it—where did the 3 and the $\frac{3}{8}$ come from?" Write an explanation that could answer Brandon's question.

3:10

3:10 Alice forgot what 7×8 equals. Alice knows that $5 \times 8 = 40$ and $2 \times 8 = 16$.

(1) Write a sentence to tell Alice how she can find the value of 7×8 by using the two facts she knows.

(2) Draw a diagram that could help Alice understand why your method works.

(3) Choose two numbers other than 7 and 8, and try using your method to multiply them. Will your method work for any pair of factors? Say why you think so.

In earlier grades, task **5:1 Juice Box Mixup** is a problem in context with an interpretation in terms of multiplication distributing over subtraction; tasks **4:5 Fraction Products and Properties**, **5:7 Shipwrecks**, and **5:14 Brandon's Equation** involve fraction products that could be evaluated using the distributive property; and task **3:10 Alice's Multiplication Fact** involves using the distributive property as a calculation strategy for a product of two one-digit numbers.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:12 Coordinate Triangle

Teacher Notes

Central math concepts

After studying length as a measurable quantity in the primary grades, students in upper elementary grades learn to recognize area as a measurable attribute of plane figures. Area is the amount of two-dimensional surface a figure contains. Consistent with this idea, congruent figures are assumed to enclose equal areas. Upper-elementary grades students also learn the concepts involved in measuring area (CCSS 3.MD.C.5):

- **A unit of measure for area**: An area unit is built from a chosen length unit. Given a length unit, a square with side length equal to 1 unit, called "a unit square," is said to have "one square unit" of area.
- **Quantifying area**: A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Because a rectangle with whole-number side lengths of L units and W units can be tiled with an L-by-W array of unit squares, the area of such a rectangle equals the product $L \times W$. As students learn to multiply fractions, they understand that even when a rectangle has fractional side lengths of L units and W units, the area of the rectangle still equals the product $L \times W$.[†]

From these beginnings,

> Using the shape composition and decomposition skills acquired in earlier grades, students [in grade 6] learn to develop area formulas for parallelograms, then triangles. They learn how to address three different cases for triangles: a height that is a side of a right angle, a height that 'lies over the base' and a height that is outside the triangle.
>
> Through such activity, students learn that any side of a triangle can be considered as a base and the choice of base determines the height (thus, the base is not necessarily horizontal and the height is not always in the interior of the triangle). The ability to view a triangle as part of a parallelogram composed of two copies of that triangle and the understanding that area is additive … provides a justification for halving the product of the base times the height, helping students guard against the common error of forgetting to take half.
>
> *Progression document*, p. 19.[‡]

Triangles on the same base, and with the same height relative to that base, all have equal areas, even though their shapes look quite different. (See an animation illustrating this.) This principle, which is involved in part (2) of task 6:12, can be seen as a consequence of the formula $A = \frac{1}{2}\,b\,h$.

More directly, consider the first figure, in which parallelograms ABCD and ABEF are on the same base and between the same parallels. It can be shown using triangle congruence theorems that triangles ADF and BCE are congruent, so they have equal areas, and hence the area of ABED less the area of BCE equals the area of ABED less the area of ADF. Therefore the

> 6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

Answer

(1) 21 square units. **(2)** The area does not change.

Click here for a student-facing version of the task.

Refer to the Standards

6.G.A.1, 3; MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

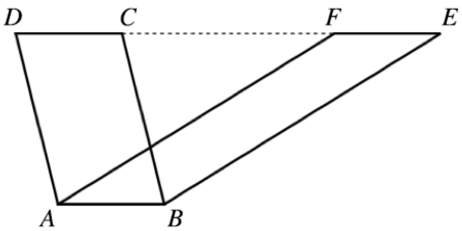
Aspect(s) of rigor:

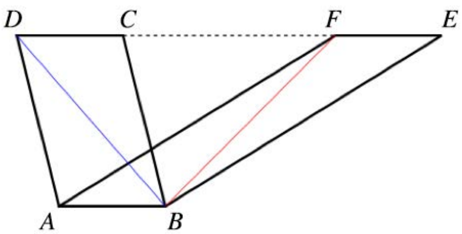
Concepts, Procedural skill and fluency

Additional notes on the design of the task

The task does not include a diagram, because mapping the given information into the coordinate setting is part of the work.

areas of the parallelograms are equal. And since the triangles ABD and ABF are half the parallelograms, the areas of the triangles on the same base and with equal heights are equal.





Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: multiplying by $\frac{1}{2}$; remembering a multiplication fact; graphing points on a coordinate plane; and understanding signed numbers on the coordinate plane.

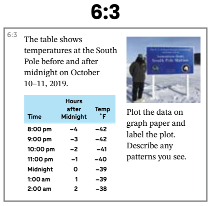
Extending the task

How might students drive the conversation further?

- Students could find a value of x such that the triangle with vertices (1, 2), (-5, 2), and (x, 9) is a right triangle.
- Students could find the coordinates of a point that makes a parallelogram with the given vertices of the triangle.

Related Math Milestones tasks

6:3

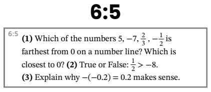


6:3 The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

Time	Hours after Midnight	Temp °F
8:00 pm	–4	–42
9:00 pm	–3	–42
10:00 pm	–2	–41
11:00 pm	–1	–40
Midnight	0	–39
1:00 am	1	–39
2:00 am	2	–38

Plot the data on graph paper and label the plot. Describe any patterns you see.

6:5



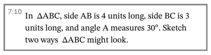
6:5 **(1)** Which of the numbers 5, -7, $\frac{2}{3}$, $-\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$.
(3) Explain why $-(-0.2) = 0.2$ makes sense.

Task **6:3 South Pole Temperatures** involves positive and negative coordinates in the coordinate plane, and task **6:5 Positive and Negative Numbers** involves signed rational numbers on a number line.

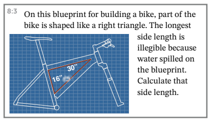
Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:12? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:12? In what specific ways do they differ from 6:12?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:12 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

7:10

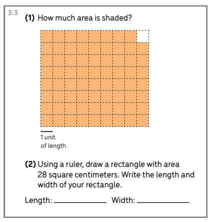

7:10 In ΔABC, side AB is 4 units long, side BC is 3 units long, and angle A measures 30°. Sketch two ways ΔABC might look.

8:3


8:3 On this blueprint for building a bike, part of the bike is shaped like a right triangle. The longest side length is illegible because water spilled on the blueprint. Calculate that side length.

In later grades, task **7:10 Triangle Conditions** advances beyond area measurement to analyze length and angle measures in a triangle. Task **8:3 Bicycle Blueprint** involves the Pythagorean theorem, which among other things enables distances between two points to be calculated when the points don't share a common *x*-coordinate or a common *y*-coordinate.

3:3

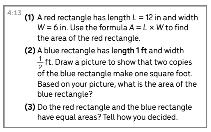

3:3 **(1)** How much area is shaded?

1 unit of length

(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: ________ Width: ________

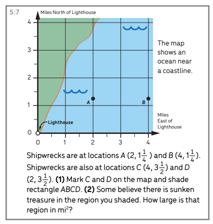
4:13


4:13 **(1)** A red rectangle has length $L = 12$ in and width $W = 6$ in. Use the formula $A = L \times W$ to find the area of the red rectangle.

(2) A blue rectangle has length **1 ft** and width $\frac{1}{2}$ ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?

(3) Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.

5:7



Shipwrecks are at locations $A\ (2, 1\frac{1}{4})$ and $B\ (4, 1\frac{1}{4})$. Shipwrecks are also at locations $C\ (4, 3\frac{1}{2})$ and $D\ (2, 3\frac{1}{2})$. **(1)** Mark C and D on the map and shade rectangle $ABCD$. **(2)** Some believe there is sunken treasure in the region you shaded. How large is that region in mi^2?

In earlier grades, tasks **3:3 Length and Area Quantities** and **4:13 Area Units** involve concepts of area measurement. Task **5:7 Shipwrecks** involves the area of a rectangle specified by four points in the coordinate plane.

† An example from the NF *Progression* document: "Instead of using a unit square with a side length of 1 inch or 1 centimeter, fifth graders use a unit square with a side length that is a fractional unit. For example, a 5/3 by 1/2 rectangle can be tiled by 30 unit squares with side length 1/6. Because 36 of these unit squares tile a 1 by 1 square, each has area 1/36. So the area of the rectangle is 30 thirty-sixths, which is 5/3 × 1/2, the product of the side lengths." (p. 18)

‡ Common Core Standards Writing Team. (2013, September 19). Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Geometry. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:13 Is There a Solution? (Multiplication)

Teacher Notes

Central math concepts

An equation can be viewed as a question: Which values from a specified set, if any, make the equation true? Solving an equation is a process of reasoning resulting in a complete answer to that question.

Task 6:13 explores the extension of the number system from whole numbers to fractions. If fractions didn't exist, we would have to invent them. Consider the dilemma of a third-grade student who solves problems like $72 \div 9 = 8$ but who may wonder what to make of a problem like $72 \div 10 = ?$. The unknown factor can't be 7 (because $10 \times 7 = 70$, which is too small); and the unknown factor can't be 8 (because $10 \times 8 = 80$, which is too large). So maybe the unknown factor is between 7 and 8? Closer to 7, because 70 is closer to 72 than 80 is? It isn't until the upper elementary grades that students fully integrate positive fractions into their expanding system of numbers and operations, and find solutions to equations like $72 \div 10 = ?$, or indeed $241p = \frac{3}{4}$.

The questions in task 6:13 could be settled procedurally by dividing both sides of the equation by 241 and then calculating the value of the quotient $\frac{3}{4} \div 241 = \frac{3}{964}$. However, the task does not ask for the exact value of p. The task thereby aims at non-procedural skills, especially the algebraic skill of looking for and making use of structure (CCSS MP.7), which grows in importance throughout students' study of algebra. And the task prioritizes non-procedural knowledge, like knowing how the size of a product depends on the size of its factors. If we can view the equation $241p = \frac{3}{4}$ as asking a question, the question might read like this: "I am thinking of a number. 241 times as much as my number equals $\frac{3}{4}$. What is my number?" This rendering of the equation emphasizes the sizes of the quantities involved and the meaning of the operation involved.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: the written conventions of algebra, specifically the conventional omission of the multiplication symbol in a product; understanding of how the size of a product depends on the size of its factors; understanding division as an unknown factor problem; and solving one-step equations.

Extending the task

How might students drive the conversation further?

- Students could ask or be asked whether it is mathematically possible to write an equation of the form $ap = b$ that has no solution even among the fractions. (Do not assume that a and b are necessarily whole numbers, but assume that a is nonzero.) As part of this discussion,

> 6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

Answer

There is no whole number that solves the equation. There is a non-whole number that solves the equation. (Reasoning for these decisions may vary.)

Click here for a student-facing version of the task.

Refer to the Standards

6.EE.B.5; MP.1, MP.3, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Additional notes on the design of the task

- The intent of saying "pencils down" is to invite a conceptual approach.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:13? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 6:13? In what specific ways do they differ from 6:13?

students could consider what happens when both sides of the equation are multiplied by $\frac{1}{a}$. (Recall from the properties of operations that for every nonzero number a, there exists a number $\frac{1}{a}$ such that $\frac{1}{a} \times a = 1$. Recall too from the properties of operations that $1 \times q = q$ for every number q.)

- Students could ask or be asked whether it is mathematically possible to write an equation of the form $ax = b$ that has two different solutions.
- Students could make sense of the equation $241p = \frac{3}{4}$ by creating word problems in which the answer is the solution to the equation. (For example, "A stack of 241 sheets of paper was measured to be $\frac{3}{4}$ in thick. How thick, measured in inches, is one sheet of paper?")

Related Math Milestones tasks

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

7:5

7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

5:2

5:2 After a hurricane, the 12 residents of a nursing home didn't have any clean water to drink. Their neighbors donated 40 gallons of bottled water, which would provide ____ gallons for each resident.

Task **6:1 Charging Cord** is a word problem for which a natural equation model ($\frac{2}{3}x = \frac{1}{2}$) also involves a product that is less than the coefficient.

In later grades, task **7:5 Is There a Solution? (Addition)** involves the next major extension of the number system, from the fractions to the rational numbers.

In earlier grades, task **5:2 Water Relief** involves the division of two whole numbers leading to a fractional quotient.

Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:13 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

6:13 Is There a Solution? (Multiplication)

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

6:14 Dividing Decimals and Fractions

Teacher Notes

Central math concepts

Task 6:14 focuses on procedures. The two types of problems considered—calculating the quotient of two decimals, and calculating the quotient of two fractions—frequently arise in the course of solving problems. For both problem types, an efficient pencil-and-paper algorithm exists.

> **Computation algorithm**. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.
>
> **Computation strategy**. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Algorithms are usefully distinguished from strategies (CCSS, p. 85; see figure). Strategies are "purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another." Mental calculation often uses strategies. For example, we could calculate 15 × 12 by thinking of 3 × (5 × 12) = 3 × 60, or by thinking of 10 × 12 + 5 × 12, or in numerous other ways. As another example, we could calculate $\frac{2}{3} \div \frac{3}{4}$ by thinking of $\frac{8}{12} \div \frac{9}{12}$ and thinking in units of twelfths to see that $\frac{8}{12} \div \frac{9}{12}$ is 8 ÷ 9 or $\frac{8}{9}$. Flexible procedural fluency based on place value, properties of operations, relationships between operations, and ideas of equivalence is valuable both in practical terms and also as a way of doing arithmetic that prefigures algebra. In algebra, strategies are the most common way of working.

Algorithms are inflexible by definition. One step follows another in the prescribed order. Some algorithms are also much simpler to execute than others. The standard multi-digit addition algorithm is less complex than the standard multi-digit subtraction algorithm. The standard algorithm for dividing fractions is less complex than the standard algorithm for dividing multi-digit numbers.

An important value in mathematics education is that of being able to solve problems in multiple ways. This brings the pleasures of seeing how a coherent subject holds together, and it allows students to check answers, unify their understanding of concepts, and learn from different ways of thinking that emerge in the classroom community. A parallel but also important outcome of mathematics education is for students to be supported in gaining procedural fluency with algorithms for the actually quite small set of recurrent problem types for which an algorithm exists. (This small set can be found in the CCSS-M by searching for "algorithm.") With division of fractions and decimals in grade 6, students who began learning to say the counting words in kindergarten reach the culmination of many interwoven learning progressions in the procedures for adding, subtracting, multiplying, and dividing whole numbers, fractions, and decimals.

> 6:14 *Pencil and paper* **(1)** 81.53 ÷ 3.1 = ? **(2)** $\frac{7}{8} \div \frac{2}{3}$ = ? **(3)** Check both of your answers by multiplying.

Answer

(1) 26.3. **(2)** $\frac{21}{16}$. **(3)** See image.

```
   26.3
 ×  3.1
 ------
    263
   7890
 ------
  81.53
```

$$\frac{2}{3} \times \frac{21}{16} = \frac{2 \times 21}{3 \times 16} = \frac{42}{48} = \frac{7}{8}$$

Click here for a student-facing version of the task.

Refer to the Standards

6.NS.A.1, 6.NS.B; MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency

Additional notes on the design of the task

Checking the answers to parts (1) and (2) by multiplying offers additional procedural practice and reinforces the relationship between multiplication and division: *C* ÷ *A* is the unknown factor in $A \times \square = C$.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: place value, number sense of decimals, and decimal notation; number sense of fractions; multiplication facts and related quotients; and the relationship between multiplication and division.

Extending the task

How might students drive the conversation further?

- Students could make sense of their quotients another way by making estimates of the values; for example, 81.53 ÷ 3.1 ≈ 81 ÷ 3 = 27 which compares well to 26.3, or $\frac{7}{8} \div \frac{2}{3} \approx 1 \div \frac{2}{3} = \frac{3}{2}$ which compares well to $\frac{21}{16}$ because $\frac{3}{2} = \frac{21}{14}$.
- Students could enter $(\frac{7}{8})/(\frac{2}{3})$ into a calculator, obtaining the result 1.3125. Use pencil and paper to multiply this number by 16. What is the result? Does the result make sense?

Related Math Milestones tasks

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1-1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9+9)$.
(2) Show calculation 1(a) on a number line.

8:6

8:6 Write as a fraction in lowest terms: **(1)** $1.041\overline{6}$. **(2)** $3^2 \cdot 3^{-5}$.

5:5

5:5 Write the requested values.

4087 × 53 = ?	$\frac{1}{10} \div 10 = ?$	0.4 × 0.9 = ?
246 × 914 = ?	$\frac{7}{8} \times \frac{5}{3} = ?$	0.75 ÷ 0.01 = ?
9744 ÷ 12 = ?	8 × ? = 73	0.63 ÷ 0.3 = ?
1461 ÷ 6 = ?	$3 \div \frac{1}{8} = ?$	0.86 + 0.4 = ?
4 − (8 − 4) = ?	$\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$	0.72 − 0.17 = ?
	$\frac{1}{3} \div (6 \times 5) = ?$	0.02 + 0.2 = ?
		0.8 − 0.55 = ?
		637 − 1.31 = ?

Tasks **6:1 Charging Cord** and **6:9 Truckload of Gravel** involve fraction quotients in context. Task **6:13 Is There a Solution? (Multiplication)** considers an unknown factor problem as an algebraic equation.

In later grades, task **7:9 Calculating with Rational Numbers** involves extending fraction procedures to the rational number system. Task **8:6 Rational Form** (part (b)) involves fraction calculations expressed in the notation of positive and negative exponents.

In earlier grades, task **5:5 Calculating** includes a range of procedural tasks, some perhaps more amenable to strategies and others perhaps more amenable to algorithms.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 6:14? Locate 2–3 similar tasks in that unit. How are the tasks similar to each other, and to 6:14? In what specific ways do they differ from 6:14?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 6:14 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

6:14 Dividing Decimals and Fractions

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?