7:10 Triangle Conditions

Teacher Notes

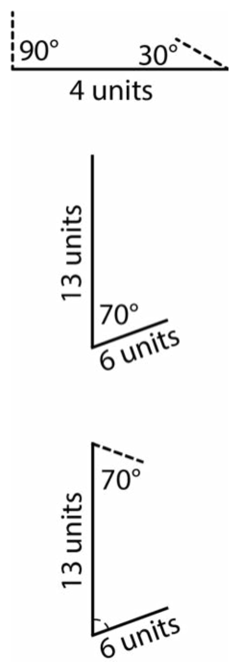
Math Milestones™

Central math concepts

Imagine that a student comes to you with the following proposal: "Let's each draw a square in the coordinate plane. Both of our squares must have a perimeter of 28 units. You go first." Accepting this curious proposal, you begin by noting to yourself that a square with a perimeter of 28 units necessarily has a side length of 7 units. You draw your square and wait for the student to draw their square. At this point you don't know if the student will draw their square near yours, or if their square will be rotated relative to yours. What you do know, however, is that whatever square they draw, it will be congruent to yours. One could say that knowing the perimeter of a square determines the square "up to congruence."

By contrast, knowing the perimeter of a triangle doesn't determine the triangle up to congruence. For example, if the student's proposal is to draw triangles that must have a perimeter of 240, then you might draw a 15-112-113 right triangle while the student might draw a 40-96-104 right triangle. These triangles aren't congruent.

Perimeter doesn't determine a triangle up to congruence, but specifying some kinds of information does determine a triangle up to congruence. Important cases of this include specifying all three side lengths; specifying two angles and the included side (illustrated in the first case shown in the figure); and specifying two sides and the included angle (illustrated in the second case shown in the figure). In high school, students learn to prove these claims deductively, whereas in grade 7 the analysis of such given conditions is not an axiomatic-deductive process. The intuitions and facts that students acquire, however, are useful even in the shorter term, as when the Angle-Angle Similarity criterion for triangles plays a role in the concept of the slope of a line in the coordinate plane in grade 8. The Angle-Angle Similarity criterion, in turn, depends on the Angle-Side-Angle criterion for triangle congruence, an early glimpse of which is visible in the figure, top.

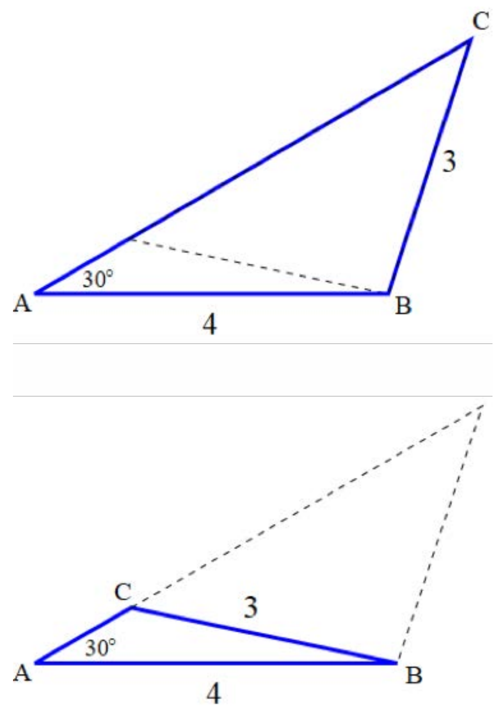


Some conditions for a triangle are impossible to satisfy. This is illustrated in the third case in the figure: there is no triangle with adjacent sides of 6 units and 13 units, and the 70° angle shown. For more discussion of triangle conditions and geometry in grade 7, see the Progression document for 7–8 and High School Geometry† and in the Progression document for 6–8 Expressions and Equations.‡

7:10 In ΔABC, side AB is 4 units long, side BC is 3 units long, and angle A measures 30°. Sketch two ways ΔABC might look.

Answer

See the figure.



Click here for a student-facing version of the task.

Refer to the Standards

7.G.A.2; MP.3, MP.5, MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: using construction tools, including technology; recognizing, measuring, and creating angles; using geometry notation; and constructing mathematical arguments.

Extending the task

How might students drive the conversation further?

- Students could see if there are two different triangles if the measure of angle *A* in the task is specified as 90°.
- Students could see if there are two different triangles if the measure of angle *A* is specified as any positive angle less than 20°.

Related Math Milestones tasks

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

8:11

8:11 Study a proof of the Angle-Angle criterion for triangle similarity. Explain one step of the proof in your own words.

6:12

6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (−5, 2), and (−8, 9)? **(2)** How does the area change if we change the third vertex to (−3, 9)?

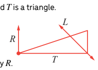
5:8

5:8 A *scalene triangle* is a triangle in which the sides all have different lengths. Thinking about this, Alana decided there should also be a name for quadrilaterals in which the sides all have different lengths. She said, "I'll name them after myself." She defined an *alana-gon* to be a quadrilateral in which the four sides all have different lengths. **(1)** Draw an example of an alana-gon. **(2)** True or false: **(a)** All squares are alana-gons. **(b)** No trapezoids are alana-gons.

4:8

4:8 *L* is a line, *R* is a ray, and *T* is a triangle. True or false:

(1) Line *L* is a line of symmetry for triangle *T*.



(2) Line *L* intersects ray *R*.

(3) Triangle *T* has two angles measuring less than 90 degrees.

An implication of task **7:13 Wire Circle** is that the circumference of a circle is enough information to determine the diameter of the circle (and therefore enough information to determine the circle, up to congruence).

In later grades, task **8:11 Angle-Angle Similarity Proof** focuses on a criterion establishing that if two angle measures are specified in a triangle, then although the triangle isn't specified up to congruence, it is specified up to similarity.

In earlier grades, task **6:12 Coordinate Triangle** specifies a triangle not with length and angle information, but with coordinate values for the vertices. In task **5:8 Alana's Shape Category**, a condition specifies not a single shape but an entire category of non-congruent shapes. Angles and angle measure are involved in task **4:8 Shapes with Given Positions**.

Additional notes on the design of the task

- The task does not include a diagram, because the product the task calls for is a diagram.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:10? Locate 2–3 similar tasks in that unit. How are the tasks similar to each other, and to 7:10? In what specific ways do they differ from 7:10?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:10 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 7–8, HS, Geometry*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

‡ Common Core Standards Writing Team. (2011). *Progressions for the Common Core State Standards for Mathematics: 6–8, Expressions and Equations* (Draft, 4/22/2011), p. 12. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

7:10 Triangle Conditions

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?