7:11 Ticket Offers

Teacher Notes

Central math concepts

Website A and Website B are both subtracting a dollar amount from the theater price, but the dollar amount subtracted by Website A is constant, whereas the dollar amount subtracted by Website B is proportional to the theater price. The smaller the theater price, the smaller the dollar amount subtracted by Website B. Therefore if the theater price is low enough, the dollar amount subtracted by Website B will be less than the dollar amount subtracted by Website A, and Website A will be the better deal. Conversely if the theater price is high enough, the dollar amount subtracted by Website B will be greater than the dollar amount subtracted by Website A, and Website B will be the better deal. The "hidden variable" in task 7:7 is the theater price, which is not given as a number. A breakthrough in this task is to realize that the theater price *is* a variable.

Situations commonly arise that involve a choice between an absolute dollar amount and a percentage. For example, one grocery store might be selling birthday cakes at half-price, while a nearby grocery store might be offering $10 off the price of a birthday cake. Or, when a composer sells a song to a film production company, the composer might have a choice between receiving a fixed dollar amount for the song versus receiving a percentage of the profits generated by the film. Just as the better deal in task 7:11 depends on the theater price of the ticket, the better deal for the cake buyer or the composer depends on estimating the price of a birthday cake or the profit potential of a forthcoming film.

If the quantitative relationships in task 7:11 were expressed algebraically, one could write $A = t - 7.5$ and $B = t - 0.25t$, where t is the theater price in dollars, A is the final cost on Website A in dollars, and B is the final cost on Website B in dollars. Using the distributive property, function B could be rewritten as $B = t(1 - 0.25)$ or, after simplifying, $B = 0.75t$. The price at which the two websites offer the same deal could be found by solving the equation $t - 7.5 = 0.75t$. The function equations $A = t - 7.5$ and $B = 0.75t$ both define linear functions.

Relevant prior knowledge

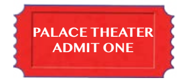
The following mathematics knowledge may be activated, extended, and deepened while students work on the task: finding percent of a total; mental calculation; and using fraction-decimal-percent equivalents.

Extending the task

How might students drive the conversation further?

- Students might realize that there exists a particular value of the theater price for which Website A and Website B are offering the ticket at the same final cost. (Or students might ask about this, or they could be

7:11 Nechama is shopping online for a ticket to a play. Website A offers a discount of $7.50 off the theater price. Website B offers a discount of 25% off the theater price. **(1)** Is it mathematically possible that Website A is a better deal than Website B? **(2)** Is it mathematically possible that Website B is a better deal than Website A? *Prove your answers.*



Answer

(1) Yes. Proof: The theater price could be $10, in which case Website A is offering the ticket at a final cost of $2.50, which is a better deal than Website B, which is offering the ticket at a final cost of $7.50. **(2)** Yes. The theater price could be $100, in which case Website B is offering the ticket at a final cost of $75, which is a better deal than Website A, which is offering the ticket at a final cost of $92.50.

Click here for a student-facing version of the task.

Refer to the Standards

7.RP.A.3, 7.EE.B; MP.2, MP.3, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- Theater tickets tend to vary widely in price. Students may want to discuss and agree upon a reasonable range of theater prices to consider, for example $10 to $50.

asked about it.) What are some ways of determining this “crossover” value? Consider using tables, graphs, and/or equations.

- Students could compare cases in which giving a single example does, or does not, prove a statement conclusively. For example, suppose the statement to be proved is, “Given any two even whole numbers, their product is even.” Does the example 8 × 6 = 48 prove the statement? Would a hundred specific examples prove the statement? If not, what sort of argument would prove the statement conclusively?

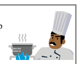
Related Math Milestones tasks

7:1

7:1 The cost of a phone is the phone’s price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn’t cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let’s use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?



8:7

8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

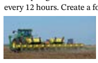
(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

6:2

6:2 **(1)** Would you prefer 33% of a \$100 prize or 75% of a \$50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a \$16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a \$100 item adds ___ dollars to the cost. A 3% tax on a \$1 item adds ___ dollars to the cost.

6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.



6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I’ll use ___ gal. **(b)** If I drive 5,700 mi, I’ll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I’ll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Task **7:1 Phone Cost** uses percent in a problem about algebraic expressions and the distributive property.

In later grades, tasks **8:9 Water Evaporation Model** and **8:7 Flight Times and Distances** prominently feature linear functions, which formalize the quantitative relationships underlying task 7:11.

In earlier grades, task **6:2 Prizes, Prices, and Percents** prominently features percent calculations involving dollar amounts. Tasks **6:6 Planting Corn** and **6:4 Gas Mileage** prominently feature unit rates and proportional relationships.

- The phrasing of the task is mathematically imprecise in the way that everyday language is mathematically imprecise. Specifically, the “better deal” between the two websites is intended to refer to the offer with the lower final cost after the discount is subtracted from the theater price. This meaning could be made explicit through discussion or partner work (especially if students aren’t very familiar with purchases, discounts, and deals).

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:11? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:11? In what specific ways do they differ from 7:11?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?