7:14 Comparing Rose’s and Liba’s Solutions

Teacher Notes

Math Milestones™

Central math concepts

The essential distinction between Rose’s and Liba’s solutions to the balloon problem is that Rose’s solution is *arithmetic* whereas Liba’s solution is *algebraic*. Mathematician and mathematics educator Roger Howe, who first used the balloon problem to illustrate this distinction, described it this way:

> “One solution is arithmetic, in the sense that it uses no variables, but simply makes a succession of calculations based on the information in the problem. The other is algebraic. It defines variables, creates expressions and formulates equations in those variables. Then it solves the equations using the standard moves of elementary algebra.” (p. 1)
>
> Roger Howe, “From Arithmetic to Algebra”

The arithmetic and algebraic solutions are both mathematically valid. For problems that aren’t very complex, the arithmetic approach might be faster and more reliable than the algebraic approach, to the extent that arithmetic is settled knowledge whereas algebra is a new field of study in middle school. For more complex problems, however, the algebraic approach might be the only approach that successfully unravels the problem to find the solution. Algebra can be used to solve problems and create mathematical models for situations where arithmetic alone won’t suffice. So it is important for middle-grades students’ mathematical futures that they make the transition from arithmetic to algebra. In task 7:14, a comparison of arithmetic and algebraic approaches creates opportunities for reflecting on that transition.

Comparing the two approaches sheds light on both. For example, observe that the operations Liba used to *build* an equation are the reverse of the operations used by Rose. Rose first subtracted, then divided; whereas Liba built an equation by first multiplying, then adding. But then observe that the operations Liba used to solve the equation were the same as the operations by Rose: subtraction, then division. Another thing to notice is that Rose likely calculated 24 for a reason, namely to find out how many balloons are packaged up. By contrast, in Liba’s approach the balloon problem can be solved without ever thinking about that particular quantity in the situation: the 24 could just arise as part of a procedural step when carrying out the standard moves of solving an equation. In that sense, the arithmetic solution could be said to probe the quantities in the situation more attentively. Therefore both approaches have mathematical depth and reflect problem-solving power, though the algebraic approach is the one that points the way toward the further mathematics of grade 8 and high school.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: basic meanings of the

7:14 Rose and Liba both solved this problem: *Jannat has 4 packs of balloons and 5 single balloons—29 balloons in all. How many balloons are in a pack?* Explain both of Rose’s steps. Check that Liba’s equations are all true statements about the balloons.

Rose	**Liba**
$29 - 5 = 24$	Let x be the # of balloons in a pack.
$24 \div 4 = 6$	$4x + 5 = 29$
	$4x = 24$
	$x = 6$

Answer

(1) Explanations of Rose’s steps will vary but should include: a reason or purpose why Rose subtracted 29 - 5; what quantity in the situation the result 24 refers to; a reason or purpose why Rose divided 24 ÷ 4; and what quantity in the situation the result 6 refers to. **(2)** Liba’s first equation is a true statement about the balloons because if you multiply the number of balloons in a pack (4) by the number of packs (x), then add the 5 single balloons, you get the total number of balloons (29). Liba’s second equation is a true statement about the balloons because it follows from the first equation that if you take away the 5 single balloons, the balloons in packs will total 24. Liba’s third equation is a true statement about the balloons because it follows from the second equation that the unknown factor must be 6.

Click here for a student-facing version of the task.

Refer to the Standards

7.EE.B.4; MP.2, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

operations of addition, subtraction, multiplication, and division; and mental calculation.

Extending the task

How might students drive the conversation further?

- Students could create an equation of the same form as in task 7:14, such as $3x - 5 = 2$, create a corresponding word problem, and then solve the word problem using both the algebraic and arithmetic approaches.

Related Math Milestones tasks

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

8:1

8:1 Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. Assuming a linear model, about how many more hours will it take him to finish summarizing?

8:2

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

Task **7:13 Wire Circle** involves a situation with quantitative relationships analogous to the balloon problem, which means that task 7:13 could be used to anchor a discussion like the one comparing Rose's and Liba's solutions.

In later grades, task **8:1 Xavier's Notes** involves a situation in which arithmetic and algebraic approaches are both available. Task **8:2 Pottery Factory** involves an opportunity to build and solve a constraint equation.

In earlier grades, some students might approach task **6:1 Charging Cord** algebraically while others might approach the task arithmetically; the same could be said of task **6:9 Truckload of Gravel**.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- Task 7:14 has a problem-within-a-problem structure that may be unfamiliar. Students could potentially warm up for task 7:14 by first solving the balloon problem.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:14? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:14? In what specific ways do they differ from 7:14?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:14 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?