7:1 Phone Cost

Teacher Notes

Central math concepts

Two expressions are called *equivalent* if the two expressions name the same number regardless of which value is substituted into them. One way to prove that two expressions are equivalent is to apply properties of operations to transform one of the expressions into the other. For example, to prove that the expression $ac + ad + bc + bd$ is equivalent to the expression $(a + b)(c + d)$, one could begin by applying the distributive property to the first two terms of the first expression, resulting in $a(c + d) + bc + bd$. Next, one could apply the distributive property to the last two terms, resulting in $a(c + d) + b(c + d)$. Finally, one could apply the distributive property to the two addends $a(c + d)$ and $b(c + d)$, resulting in $(a + b)(c + d)$. Because the expression $ac + ad + bc + bd$ can be transformed into $(a + b)(c + d)$ using properties of operations, the two expressions must name the same number regardless of which value is substituted into them. (If any exception existed, then that exception would also be an exception to the properties of operations, but the properties of operations are true for all numbers without exception.)

In task 7:1, the distributive property can be used to transform the expression $P + 0.0625*P$ into the expression $P*1.0625$. This shows that the two expressions will always give the same result for the total cost, regardless of the before-tax price. This is just one example of how algebra allows us to make statements or draw conclusions that refer to infinitely many cases.

An expression records operations with numbers and with letters standing for numbers. The expression in part (1) records the calculation "Multiply P by 0.0625 then add the result to P." The expression in part (2) records the calculation "Multiply P by 1.0625." The fact that one expression involves addition while the other involves multiplication is a hint that the distributive property is involved. This is because the mathematical relationship between multiplication and addition is given by the distributive property.†

The observation that multiplying P by 0.0625 then adding the result to P is equivalent to multiplying P by 1.0625 foreshadows the mathematics of exponential functions. In high school mathematics courses, students take advantage of the equivalence between $P + rP$ and $(1 + r)P$ to create exponential functions. For example, in compound interest we might consider that after the first interest period, an amount of principal P has increased in value to $P + rP = (1 + r)P$; after the second interest period, the principal has increased in value to

$$
\begin{aligned}
&(1 + r)P + r((1+ r)P) \\
&= (1 + r)((1 + r)P) \\
&= (1 + r)^2P,
\end{aligned}
$$

and so on, leading to consideration of the expression $(1 + r)^nP$ which defines an exponential function of the variable n.

> 7:1 The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

Answer

(1) In dollars, the cost is 264 + 0.0625*264 = 264 + 16.50 = 280.50. **(2)** In dollars, the cost is 264*1.0625 = 280.50. **(3)** Applying the distributive property, $P + 0.0625*P = 1*P + 0.0625*P = (1 + 0.0625)*P = 1.0625*P$.

Click here for a student-facing version of the task.

Refer to the Standards

7.RP.A.3, 7.EE.A; MP.2, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency, Application

Additional notes on the design of the task

- By convention, multiplication symbols are frequently omitted in algebraic expressions, but multiplication symbols are not omitted here, so as to make the role of multiplication in the task more explicit. The symbol is the one often used in technology.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: calculating tax by finding a percent of a total; and applying properties of operations to rewrite expressions in equivalent forms, especially in cases where a variable written without a coefficient must be recognized as having a coefficient of 1.

Extending the task

How might students drive the conversation further?

- Students could ask or be asked to substitute several different costs into the expressions *P* + 0.0625**P* and *P**1.0625 to verify that the expressions give the same result in different cases. Students could discuss whether the two expressions will give the same result for every possible value of the cost. If students think so, what could be a mathematical justification for that belief? For example, students could be asked to consider the expression *P* + 0.0625**P* - *P**1.0625. What does the expression simplify to?
- Students could discuss which of the two expressions is more convenient for calculating the total cost. This is an example of the way rewriting expressions in equivalent forms can serve a practical or mathematical purpose.

Related Math Milestones tasks

7:3

7:3 Write each sum as a product with the given factor. *Example:* $8 + 6x = 2 \cdot ?$ *Answer:* $8 + 6x = 2(4 + 3x)$. **(1)** $6y + 12 = 3 \cdot ?$ **(2)** $-5w + 35 = (-5) \cdot ?$ **(3)** $4z + 1 = 4 \cdot ?$ **(4)** $9ay - 9by + 27cy = (9y) \cdot ?$

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$. **(2)** Show calculation 1(a) on a number line.

7:8

7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, *P*, in millions of dollars, after pumping oil for *D* days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (*D*, *P*) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for *P* is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

8:2

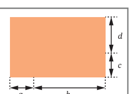
8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

6:2

6:2 **(1)** Would you prefer 33% of a $100 prize or 75% of a $50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a $16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a $100 item adds ___ dollars to the cost. A 3% tax on a $1 item adds ___ dollars to the cost.

6:11

6:11 The diagram shows a rectangle. The variables *a*, *b*, *c*, and *d* are lengths in meters.



(1) Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

Tasks **7:3 Writing Sums as Products**, **7:9 Calculating with Rational Numbers**, and **7:8 Oil Business** also involve the distributive property.

In later grades, task **8:2 Pottery Factory** involves a situation that can be modeled with an equation in which properties of operations including the distributive property would be used in the process of solving for an unknown value.

In earlier grades, task **6:2 Prizes, Prices, and Percents** involves some of the same quantities as in task 7:1 (in particular, see parts (4) and (5)). In task **6:11 Area Expressions**, students generate different expressions for the same quantity and use properties of operations to transform one of the expressions into one of the others.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:1? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:1? In what specific ways do they differ from 7:1?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Note that many familiar techniques in algebra can be understood as applications of the distributive property; for examples, see the Teacher Note for Task 7:3, Writing Sums as Products.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?