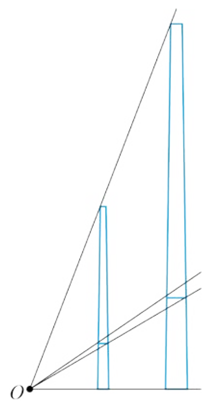
7:2 Utility Pole Scale Drawing

Teacher Notes

Central math concepts

To make a scale drawing is to make a mathematical model. A scale drawing of an object is a model of the object that accurately reflects relationships between the object's parts, in the sense that ratios of lengths measured on the drawing are equivalent to ratios of lengths measured directly on the object. For example, when measured in inches directly on the utility pole, the ratio of the two specified diameters is 9:15, and when measured in grid units on the drawing, the ratio of those same two diameters is $\frac{3}{4}$:$\frac{5}{4}$. These two ratios are equivalent; the scale factor that relates them is 12: $12 \times \frac{3}{4} = 9$, and $12 \times \frac{5}{4} = 15$. The scale factor that relates the two ratios is 12 because the measurements involved are respectively in inches and in grid units, and 1 grid unit on the drawing corresponds to 12 inches on the physical object.

Scale drawings are an intuitive application for the idea of "same shape, different size" that informs more precise concepts of geometric similarity. From the grade 8 perspective, shrinking or enlarging a scale drawing could be thought of as the effect of a dilation transformation with $r > 1$ or $r < 1$, respectively (see figure).



Dilating a scale drawing

All models simplify reality, and a scale drawing of a utility pole is a simplification in that a utility pole is three-dimensional, whereas the drawing is flat (two-dimensional). That said, the scale drawing leaves out little information by taking advantage of the rotational symmetry of the pole. There could also be situations, however, in which a three-dimensional scale model would be needed—for example, to test the effects of high winds on a tall utility pole (see figure).

Luo, Y. & Wang, Yucheng & Xie, J. & Yang, C. & Zheng, Yanfeng. (2017). Aero-elastic wind tunnel test of a high lighting pole. Wind and Structures, An International Journal. 25. 1-24.

Relevant prior knowledge

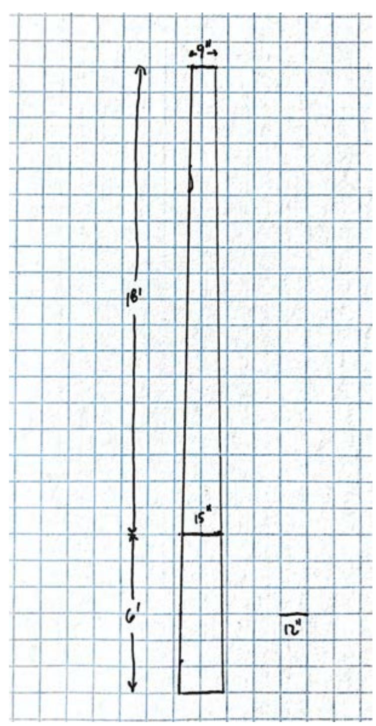
The following mathematics knowledge may be activated, extended, and deepened while students work on the task: converting between feet and inches; fraction reasoning with fourths and eighths; finding equivalent ratios; using scales and units; and applying circle measurement in context.

7:2 A utility pole 24 feet long has $28\frac{1}{4}$-inch circumference at the top and $47\frac{1}{8}$-inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale $\frac{1}{4}$ inch = 1 foot.

Answer

See example. The diameter of the utility pole at the top is 9"; in the scale drawing, the diameter is $\frac{3}{4}$ of a grid unit long. The diameter of the utility pole 6 feet from the base is 15"; in the scale drawing, this is one and a quarter ($\frac{5}{4}$) grid units.

Note: It is not required to determine the diameter of the pole at the base. The diameter at the top and the diameter 6 feet from the base can be drawn, and then the sides of the pole can be drawn starting from the ends of the top diameter, passing through the ends of the diameter 6 feet from the base, and extending to the full length of the pole.



Click here for a student-facing version of the task.

Extending the task

How might students drive the conversation further?

- Students could wonder whether (or be asked to investigate whether) the ratio

  (diameter at the top):(diameter 6 feet from the base)

  is equivalent to the ratio

  (circumference at the top):(circumference 6 feet from the base).

  How could this equivalence be explained algebraically using the formula for the circumference of a circle?

- The scale drawing could be produced using technology, as in [this example].

Related Math Milestones tasks

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

8:11

8:11 Study a proof of the Angle-Angle criterion for triangle similarity. Explain one step of the proof in your own words.

6:10

6:10 In the month of February 2021, there were 20 weekdays and 8 weekend days. Here are some questions about that month. **(1)** (Circle all of the correct answers.) The ratio of weekdays to weekend days was 20:8 10:4 5:2 5:7. **(2)** There were ___ times as many weekdays as weekend days. **(3)** True or false: $\frac{5}{7}$ of the days that month were weekdays. **(4)** Approximately what percent of the days that month were weekdays?

Task **7:13 Wire Circle** involves circle measurement in the context of finding a value that satisfies a constraint.

In later grades, task **8:11 Angle-Angle Similarity Proof** involves concepts of geometric similarity that are implicit in task 7:2.

In earlier grades, task **6:10 Weekdays and Weekend Days** involves equivalent ratios.

Refer to the Standards

7.G.A.1, 7.G.B.4; MP.4, MP.5, MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Additional notes on the design of the task

- The scale $\frac{1}{4}''$ = 1 foot may be convenient when using graph paper with quarter-inch squares. If desired, the task could be modified by specifying a different scale, such as $\frac{1}{5}''$ = 6″, which may be convenient when using graph paper with $\frac{1}{5}''$ squares.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:2? Locate 2–3 similar tasks in that unit. How are the tasks similar to each other, and to 7:2? In what specific ways do they differ from 7:2?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:2 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Math Milestones™

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?