7:3 Writing Sums as Products

Teacher Notes

Math Milestones™

Central math concepts

The mathematical relationship between multiplication and addition is given by the distributive property:

$$a(b + c) = ab + ac.$$

The distributive property is an *identity*, an equation that is true for all possible values of its variables. The left-hand side of the identity is a product of two terms (one of which is a sum), while the right-hand side of the identity is a sum of two terms (both of which are products). The distributive property allows us to rewrite sums as products and rewrite products as sums.

Many familiar techniques in algebra can be understood as applications of the distributive property:

- Distributing. When we distribute r in the example $r(k + p + 2) = rk + r(p + 2)$, we are applying the distributive property to rewrite a product of two terms as a sum of two terms. Note that we could apply the distributive property again to rewrite $r(p + 2)$ as $rp + 2r$. That would result in the identity $r(k + p + 2) = rk + rp + 2r$.
- Factoring. When we factor out y in the example $xy + yz = y(x + z)$, we are applying the distributive property to rewrite a sum of two terms as a product of two terms.
- Collecting like terms. When we collect like terms in the example $\frac{1}{8}x + -\frac{1}{2}x + \frac{1}{4}x = (\frac{1}{8} + -\frac{1}{2} + \frac{1}{4})x$, we are applying the distributive property to rewrite a sum of three terms as a product of two terms. Of course we could continue to evaluate the sum $\frac{1}{8} + -\frac{1}{2} + \frac{1}{4} = -\frac{1}{8}$, which would result in the identity $\frac{1}{8}x + -\frac{1}{2}x + \frac{1}{4}x = -\frac{1}{8}x$.

Sometimes the distributive property is described as saying that "multiplication distributes over addition." One could equally well say that multiplication distributes over subtraction: $a(b - c) = ab - ac$. This is because subtracting c is the same as adding $-c$, so the distributive property applies as usual. (In more detail, the difference $b - c$ could be rewritten as a sum, $b + -c$, and then $a(b + -c) = ab + a(-c)$ by the distributive property. And because $a(-c) = -ac$, we have $a(b + -c) = ab + -ac$, which we could rewrite finally as $a(b - c) = ab - ac$.)

Similarly, division distributes over addition and subtraction, in the sense that $(a \pm b) \div c = a \div c \pm b \div c$, where $c \neq 0$. This is because dividing by c is the same as multiplying by $\frac{1}{c}$, so the distributive property applies as usual. (In more detail, the quotient $(a \pm b) \div c$ could be rewritten as a product, $(a \pm b) \times \frac{1}{c}$, and then $(a \pm b) \times \frac{1}{c} = a \times \frac{1}{c} \pm b \times \frac{1}{c}$ by the distributive property. We could rewrite this finally as $(a \pm b) \div c = a \div c \pm b \div c$.)

Note in the above our implicit assumption that every c has an additive inverse $-c$, and every nonzero c has a multiplicative inverse $\frac{1}{c}$. Thus,

7:3 Write each sum as a product with the given factor. *Example:* $8 + 6x = 2 \cdot ?$ *Answer:* $8 + 6x = 2(4 + 3x)$. **(1)** $6y + 12 = 3 \cdot ?$ **(2)** $-5w + 35 = (-5) \cdot ?$ **(3)** $4z + 1 = 4 \cdot ?$ **(4)** $9ay - 9by + 27cy = (9y) \cdot ?$

Answer

(1) $6y + 12 = 3 \cdot (2y + 4)$. **(2)** $-5w + 35 = (-5) \cdot (w - 7)$. **(3)** $4z + 1 = 4 \cdot (z + \frac{1}{4})$. **(4)** $9ay - 9by + 27cy = 9y(a - b + 3c)$. Equivalent expressions are also correct, for example, $-5w + 35 = (-5) \cdot (w + -7)$ for part (2), $4z + 1 = 4 \cdot (1z + 0.25)$ for part (3), etc.

Click here for a student-facing version of the task.

Refer to the Standards

7.EE.A.1; MP.1, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Additional notes on the design of the task

As indicated by the example, the desired answer for each part of the task is an equation that shows the equivalence between the initial form of the expression and the factored form. So if a student answers the first part of the task with "? = $2y + 4$," that isn't wrong at all, but in that case, prompt for a fuller answer in the form $6y + 12 = 3(2y + 4)$.

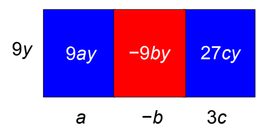
we were relying not only on the distributive property but also on other properties of operations as well. The properties work together to form a system, a system in which calculations can be made and expressions can be transformed.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: understanding of division as an unknown factor problem; the distributive property; rules for operating with signed rational numbers; and viewing expressions as objects with structure.

Extending the task

How might students drive the conversation further?



- Students could draw an area diagram to represent one of the problems (see figure).
- Students could ask or be asked if the unknown factors could be found by division. For example, if $6y + 12$ equals three times an unknown factor, then the unknown factor is $6y + 12$ divided by 3 (or equivalently $\frac{1}{3}$ times $6y + 12$).
- Students could choose values for the variables and verify that the left and right hand sides of the equations are equal when values are substituted.

Related Math Milestones tasks

7:1

7:1 The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$. **(2)** Show calculation 1(a) on a number line.

7:8

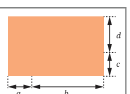
7:8 In 2018, an oil company rented an oil rig for \$100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for \$70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make \$30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

8:2

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

6:11

6:11 The diagram shows a rectangle. The variables a, b, c, and d are lengths in meters.



(1) Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

5:1

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

Tasks **7:1 Phone Cost**, **7:9 Calculating with Rational Numbers**, and **7:8 Oil Business** also involve the distributive property.

In later grades, task **8:2 Pottery Factory** involves a situation that can be modeled with an equation in which properties of operations including the distributive property would be used in the process of solving for an unknown value.

In earlier grades, task **6:11 Area Expressions** involves transforming expressions into equivalent forms using properties of operations, and task **5:1 Juice Box Mixup** has an interpretation in terms of multiplication distributing over subtraction.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:3? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:3? In what specific ways do they differ from 7:3?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:3 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

