# 7:5 Is There a Solution? (Addition)

Teacher Notes

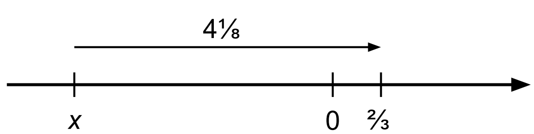
Math Milestones™

## Central math concepts

An equation can be viewed as a question: Which values from a specified set, if any, make the equation true? Solving an equation is a process of reasoning resulting in a complete answer to that question.

Task 7:5 also explores the extension of the number system from the nonnegative numbers to the rational numbers. Using only whole numbers, the equation $x + 1 = 0$ cannot be solved; but in the rational number system, any equation of the form $x + a = b$ can be solved. A key reason for this is that for any rational number $a$, there exists a rational number $-a$, called the additive inverse of $a$, for which $-a + a = 0$. In terms of this additive inverse, the solution to the equation $x + a = b$ is given by $x = b + -a$. This can be verified by substitution: the value $x = b + -a$ makes the equation $x + a = b$ true because $(b + -a) + a = b + (-a + a) = b + 0 = b$.

The questions in task 7:5 could be settled procedurally by subtracting $4\frac{1}{8}$ from both sides of the equation and then calculating the value of the difference $\frac{2}{3} - 4\frac{1}{8} = -\frac{83}{24}$. However, the task does not ask for the exact value of $x$. The task thereby aims at non-procedural skills, especially the algebraic skill of looking for and making use of structure (CCSS MP.7), which grows in importance throughout students' study of algebra. And the task prioritizes non-procedural knowledge, such as understanding addition as an operation of translation left or right along the number line, as illustrated in the figure. The figure shows that if a translation by $4\frac{1}{8}$ units from starting point $x$ ends at the point $\frac{2}{3}$ on the number line, then $x$ must be located to the left of 0 on the number line.




## Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: addition and subtraction concepts for rational numbers; number sense of the size of fractions; representing problems on a number line; understanding subtraction as an unknown addend problem; and solving one-step equations.

## Extending the task

How might students drive the conversation further?

- Students could ask or be asked whether it is mathematically possible to write an equation of the form $x + a = b$ that has no solution even among the rational numbers. As part of this discussion, students could consider

> 7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

### Answer

There is no positive number that solves the equation. There is a negative number that solves the equation. (Reasoning for these decisions may vary.)

Click here for a student-facing version of the task.

### Refer to the Standards

7.NS.A.1, 7.EE.B.4; MP.1, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

### Aspect(s) of rigor:

Concepts

### Additional notes on the design of the task

- The intent of saying "pencils down" is to invite a conceptual approach.
- Similarly, the intent of the particular numbers chosen ($4\frac{1}{8}$ and $\frac{2}{3}$) is to invite a conceptual approach.

what happens when -$a$ is added to both sides of the equation. (Recall from the properties of operations that for every number $a$, there exists a number -$a$ such that -$a$ + $a$ = 0. Recall too from the properties of operations that $q$ + 0 = $q$ for every number $q$.)

- Students could ask or be asked whether it is mathematically possible to write an equation of the form $x + a = b$ that has two different solutions.
- Students could make sense of the equation $x + 4\frac{1}{8} = \frac{2}{3}$ by creating word problems in which the answer is the solution to the equation. (For example, "The temperature outside increased by $4\frac{1}{8}$ degrees. After that, the temperature was $\frac{2}{3}$ of a degree above zero. What was the temperature before the temperature increased?")
- Students could repeat task 7:5 for an equation like $x + 4\frac{1}{8} = -2$ in which the right-hand side is negative. Without solving the equation, try to decide whether the solution to this equation is greater than or less than the solution to the original equation $x + 4\frac{1}{8} = \frac{2}{3}$.

## Related Math Milestones tasks

**7:9**

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1-1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$. **(2)** Show calculation 1(a) on a number line.

**7:12**

7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

**8:9**

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable $D$ is the depth of the soup in the pot, in units of cm, and variable $t$ is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

**6:5**

6:5 **(1)** Which of the numbers $5, -7, \frac{2}{3}, -\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$. **(3)** Explain why $-(-0.2) = 0.2$ makes sense.

**6:13**

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

Task **7:9 Calculating with Rational Numbers** includes calculations that exercise conceptual understanding of addition and subtraction of rational numbers. Task **7:12 Temperature Change** is a word problem that involves calculating with rational numbers.

In later grades, task **8:9 Water Evaporation Model** involves a linear function with a negative rate of change, putting rational number arithmetic to use for modeling a situation.

In earlier grades, task **6:5 Positive and Negative Numbers** involves concepts of absolute value and order for rational numbers on the number line. Task **6:13 Is There a Solution? (Multiplication)** involves the extension of the number system from whole numbers to fractions.

### Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:5? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:5? In what specific ways do they differ from 7:5?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:5 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

# 7:5 Is There a Solution? (Addition)

Teacher Notes

## Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

### Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

### Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

### Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?