7:6 Car A and Car B

Teacher Notes

Central math concepts

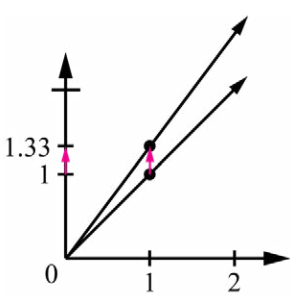
This task centers on the connections between:

- The unit rate in a proportional relationship,
- The point (1, *r*) on the graph of a proportional relationship, and
- The slope of the graph of a proportional relationship.

To appreciate these connections, first recall that the equation of a proportional relationship between two variable quantities x and y can be written as $y = rx$, where the constant r is the unit rate. This equation says that as x and y vary together, they vary in such a way that y is always r times as much as x. In particular, if $x = 1$, then we will have $y = r{\cdot}1$, or simply $y = r$. This implies that the graph of a proportional relationship includes the point $(1, r)$ where r is the unit rate. (We are assuming that 1 is a possible x-value in the given context.) Another particular case is when $x = 0$, assuming that 0 is a possible x-value in the given context; and if $x = 0$, then we will have $y = r{\cdot}0$, or simply $y = 0$. This implies that the graph of a proportional relationship includes the point (0, 0).

Therefore we could use the two points (0, 0) and $(1, r)$ to calculate the slope of the graph. The result of that calculation is $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(r - 0)}{(1 - 0)} = \frac{r}{1} = r$.

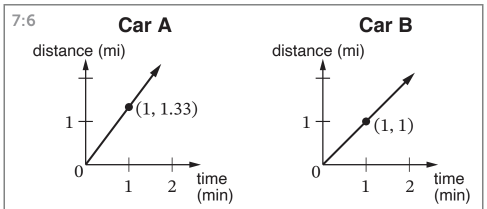
The conclusion we can draw from this calculation is an important one: the slope of the graph of a proportional relationship is the unit rate.



Notice that because the points (1, 1.33) and (1, 1) have the same x-coordinate, the point with the larger y-coordinate is necessarily located higher in the coordinate plane, and this results in that graph being the steeper of the two. This may be easier to see if the two graphs are overlaid on one another, as in the figure. This observation connects the fact that the steepness of the graph is measured by the slope with the fact that the slope is the unit rate.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: interpreting a coordinate pair in the context of a situation by referring to the two coordinate axes and the quantities they refer to in the situation; recognizing and interpreting the unit rate in a proportional relationship; relating the unit rate to an appropriate statement in ratio language (such as, "For every 3 minutes that pass, Car A travels approximately 4 miles"); and writing a two-variable equation to represent a proportional relationship.



Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in *n* minutes. **(4)** How many miles does the faster car move in 10 minutes?

Answer

(1) At the end of the first minute, Car A had gone 1.33 miles, and Car B had gone 1 mile. **(2)** Car A was moving faster than Car B. **(3)** $d = 1.33n$ (if d stands for the number of miles moved; different letters could be used for this quantity). **(4)** Using the formula, the car moves $d = 1.33(10) = 13.3$ miles in 10 minutes.

Click here for a student-facing version of the task.

Refer to the Standards

7.RP.A.2; MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Extending the task

How might students drive the conversation further?

- Students could be asked to sketch a graph for a third car (“Car C”) that is specified to be moving at a constant speed of 0.27 miles per minute. A quantitative approach to sketching the graph might be to first estimate the location of the point (1, 0.27) and draw the graph as a straight line from the origin through that point.
- Students could use a spreadsheet to enter the formula they created in part (3) and use the formula to quickly create a two-column table of *x*-*y* ordered pairs and graph the pairs of values in the table. This could be done for the slower car, as well, allowing the two graphs to be shown on the same plot.

Related Math Milestones tasks

7:8

7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company’s situation?

7:12

7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn’t cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let’s use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

8:7

8:7 City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I’ll use ___ gal. **(b)** If I drive 5,700 mi, I’ll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I’ll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Another task for grade 7 that prominently features unit rates is **7:8 Oil Business**. Task **7:12 Temperature Change** involves an average rate.

In later grades, proportional relationships are extended to linear functions. Tasks **8:9 Water Evaporation Model** and **8:7 Flight Times and Distances** prominently feature linear functions.

In earlier grades, students work with unit rates and proportional relationships. Tasks **6:6 Planting Corn** and **6:4 Gas Mileage** prominently feature proportional relationships.

Additional notes on the design of the task

- It may be productive to ask students to share with one another the reasoning that led them to choose Car A or Car B for part (2) of the task. Some students might have chosen Car A based on a visual sense of the greater steepness of its graph; other students might have reasoned numerically: a car that travels 1.33 miles in 1 minute is necessarily moving faster than a car that travels only 1 mile in 1 minute. A mathematical discussion could seek to connect those two approaches.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:6? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 7:6? In what specific ways do they differ from 7:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?