7:7 Speed Limit

Teacher Notes

Math Milestones™

Central math concepts

If we divide a measurement with units of length by a second measurement with units of time, the resulting quotient measures a quantity of a third kind, speed. And in a similar way, there are many common situations in which division creates new kinds of quantities out of the quantities being divided. For example, if we pour a quantity of sugar into warm water, then the number of grams of sugar divided by the number of liters of water provides a measure of the sweetness of the solution. If we divide the rise of a line in the coordinate plane by the run, then the resulting quotient provides a measure of the steepness of the line. If we divide the number of persons living in a county by the area of the county in square miles, then the resulting quotient measures the quantity called population density. If we divide the number of kilograms of mass in a substance by the number of cubic meters occupied by the substance, then the resulting quotient measures the quantity called mass density. The number of possible examples is endless.

Measures of speed, sweetness, population density and the like can be compared against one another to say which object is moving faster, which drink is sweeter, which county is more dense, and so on. When two measures of the same kind of quantity are given in the same units, comparing them is straightforward: a car moving at a speed of 65 mph is moving faster than a car moving at a speed of 61 mph, just as a 100-watt light bulb is brighter than a 40-watt light bulb. However, when two measures of the same kind of quantity are given in different units, their numerical values cannot be directly compared. A stick 1 meter long is not the same length as a stick 1 yard long, even though 1 = 1. In this task, we are asked to compare a speed measured in units of kilometers per hour with a speed measured in miles per hour. This invites thinking about whether a kilometer is longer or shorter than a mile, and by what factor. Quantitative literacy includes knowing a reasonable collection of such facts. No two people carry around the same quantitative knowledge of the world in their back pocket, but everyone should carry some.

Yet it is a fact of modern life that when you're connected to the internet with a web browser, you're in immediate contact with a "hive mind" that knows every fact you know and more. The expectation in task 7:7 is that students will have access to technology. What will happen when a student is given task 7:7 and, instead of doing what would have been done forty years ago, the student types "convert 100 km/hr to mph" into a browser search bar and obtains the instantaneous result 62.1371 miles per hour? What mathematical conversations are valuable at that point? Some suggestions are in "**Extending the task**."

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: converting units; working with unit rates; and using appropriate precision in applications.

> 7:7 If the speed limit in Canada is 100 km/hr and you are driving 65 mph, are you over or under the limit? By how much?

Answer

If using km/hr: You are over the speed limit by about 5 km/hr. If using mph: You are over the speed limit by about 3 mph.

Click here for a student-facing version of the task.

Refer to the Standards

7.RP.A.1; MP.5, MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Additional notes on the design of the task

- The phrasing of the task is mathematically imprecise in the way that everyday language is mathematically imprecise. This leads to the possibility of two different answers depending on the choice of units in which to measure the difference. This intentional ambiguity also leads to the possibility of different answers depending on the student's chosen level of precision. Using mathematics in real life is often less constrained, less dictated, than when mathematics is used in school. The design of this task is intended to give students a small taste of that freedom and the potential confusion such choices can give rise to.

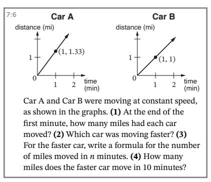
Extending the task

How might students drive the conversation further?

- If a student uses technology to perform a conversion such as 100 km/hr = 62.1371 mph (or if students are shown the output of such a technology-based calculation featuring a similar level of precision), then a worthwhile conversation might be to ask whether all of the digits to the right of the decimal point are useful. How different, in everyday terms, are the two measurements 62.1371 mph and 62.1372 mph? Additional important questions could include, “Is the computer’s value correct? How does the computer arrive at its answer?”
- Students could develop a formula that returns the speed in miles per hour given the speed in kilometers per hour. They could also develop a formula that returns the speed in kilometers per hour given the speed in miles per hour.

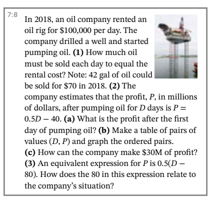
Related Math Milestones tasks

7:6

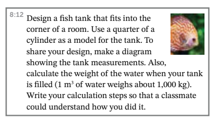

7:6
Car A / Car B

Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in *n* minutes. **(4)** How many miles does the faster car move in 10 minutes?

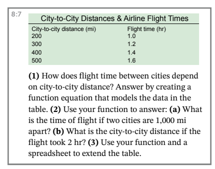
7:8


7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, *P*, in millions of dollars, after pumping oil for *D* days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for *P* is $0.5(D - 80)$. How does the 80 in this expression relate to the company’s situation?

8:12


8:12 Design a fish tank that fits into the corner of a room. Use a quarter of a cylinder as a model for the tank. To share your design, make a diagram showing the tank measurements. Also, calculate the weight of the water when your tank is filled (1 m^3 of water weighs about 1,000 kg). Write your calculation steps so that a classmate could understand how you did it.

8:7

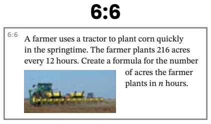

8:7

City-to-City Distances & Airline Flight Times

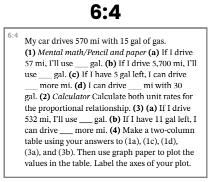
City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

6:6


6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in *n* hours.

6:4


6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I’ll use ___ gal. **(b)** If I drive 5,700 mi, I’ll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I’ll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Task **7:6 Car A and Car B** prominently features a proportional relationship in which the slope of a line is a visual measure of the speed of motion. Task **7:8 Oil Business** involves unit rates in an application context.

In later grades, task **8:12 Fish Tank Design** involves working with units and compound units. Task **8:7 Flight Times and Distances** prominently features a linear function with a rate of change formed by dividing a quantity of length by a quantity of time.

In earlier grades, tasks **6:6 Planting Corn** and **6:4 Gas Mileage** prominently feature unit rates and proportional relationships.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:7? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:7? In what specific ways do they differ from 7:7?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:7 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?