7:8 Oil Business

Teacher Notes

Central math concepts

In task 7:8, there are three distinct representations for the relationship between profit *P* and number of days *D*. The three representations are an equation, a table, and a graph. The specific question asked in part (2)(c) of the task, "How can the company make $30M of profit?" could be answered by using any of the three representations - for example, by solving the equation $0.5D - 40 = 30$ to find $D = 140$. This result shows that the company can make $30M of profit by pumping oil for 140 days. As important as it is for students to know how to establish specific facts like this, it is equally important for students to understand how each representation encodes such facts in characteristic ways:

- The graph shows that the company can make $30M of profit by pumping oil for 140 days because the graph includes the point with coordinates (140, 30).
- The table shows that the company can make $30M of profit by pumping oil for 140 days because the table includes a row with 140 in the *D* column and 30 in the *P* column.
- The equation shows that the company can make $30M of profit by pumping oil for 140 days because the result of substituting 140 for *D* in the formula $P = 0.5D - 40$ is $P = 0.5(140) - 40 = 70 - 40 = 30$.

The representations all show the same fact about the situation in different ways. As students gain fluency and confidence with algebra, the economy of the equation representation makes it primary, although graphs and tables remain valuable in many situations in school, work and life.

Representations make mathematical thinking visible, so that the thinking can be discussed, debated, and refined. Representations can also provide access points to the mathematics that rely less heavily on spoken or written language. The value of multiple representations, then, isn't primarily in providing students with multiple methods for getting answers to a given problem; it's in deepening students' grasp of the mathematics from which a given problem emerges.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: interpreting positive and negative numbers in context; substituting values into variable expressions; and using unit rates (unit price and rate per day).

Extending the task

How might students drive the conversation further?

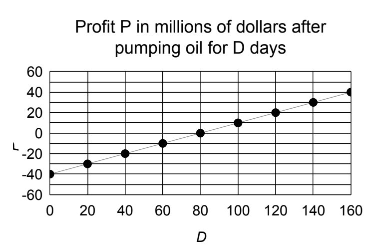
- Students could discuss the two equivalent expressions $0.5D - 40$ and $0.5(D - 80)$ in the task. What quantities in the situation are most visible in each form?

7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, *P*, in millions of dollars, after pumping oil for *D* days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (*D*, *P*) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for *P* is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

Answer

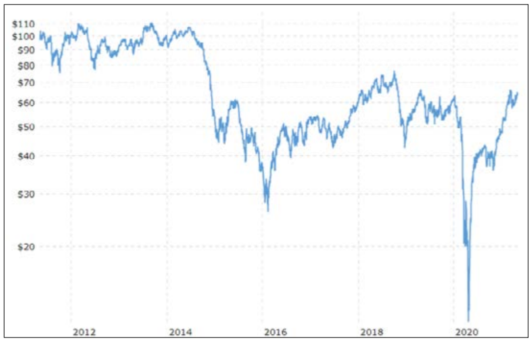
(1) 60,000 gallons per day. Answers that differ from this value because of rounding intermediate steps are acceptable. (For example, if the unit price of oil is rounded to $1.67 per gallon, then to the nearest whole number, 59,880 gallons of oil must be sold.) **(2)** **(a)** -39.5 million dollars. **(b)** Answers may vary; see example from this online spreadsheet. **(c)** Pump oil for 140 days. **(3)** After pumping oil for 80 days, the company's profit is $P = 0.5(80 - 80) = 0$. If the company pumps oil for less than 80 days, then their profit will be negative (meaning that they will lose money). If the company pumps oil for more than 80 days, their profit will be positive.

D	P
0	-40
20	-30
40	-20
60	-10
80	0
100	10
120	20
140	30
160	40



Click here for a student-facing version of the task.

- Students could ask how much the selling price of oil changes during the course of a typical year. A graph like this shows the historical trends. Is oil a risky business?



Related Math Milestones tasks

7:1

7:1 The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

7:14

7:14 Rose and Liba both solved this problem: *Jannat has 4 packs of balloons and 5 single balloons—29 balloons in all. How many balloons are in a pack?* Explain both of Rose's steps. Check that Liba's equations are all true statements about the balloons.

Rose	**Liba**
$29 - 5 = 24$	Let x be the # of balloons in a pack.
$24 \div 4 = 6$	$4x + 5 = 29$
	$4x = 24$
	$x = 6$

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

8:7

8:7 City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Like task 7:8, task **7:1 Phone Cost** involves a use of the distributive property to transform an expression in context. Task **7:14 Comparing Rose and Liba's Solutions** involves an equation like the one that could be used to answer part (2)(b) of task 7:8.

In later grades, tasks **8:9 Water Evaporation Model** and **8:7 Flight Times and Distances** prominently feature linear function models that are implicit in the company's profit model in task 7:8.

In earlier grades, tasks **6:6 Planting Corn** and **6:4 Gas Mileage** prominently feature unit rates and proportional relationships.

Refer to the Standards

7.RP.A.2b, 7.EE.A.2, 7.EE.B.4; MP.1, MP.2, MP.4, MP.5, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Additional notes on the design of the task

- Compared to the expression 0.5D - 40, the equivalent form 0.5(D - 80) is intended to make it more transparent that 80 days is the dividing line between profit and loss. To verify that 0.5(D - 80) is equivalent to 0.5D - 40, students could apply the distributive property.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:8? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:8? In what specific ways do they differ from 7:8?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:8 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Math Milestones™

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?