7:1 Phone Cost

Teacher Notes

Central math concepts

Two expressions are called *equivalent* if the two expressions name the same number regardless of which value is substituted into them. One way to prove that two expressions are equivalent is to apply properties of operations to transform one of the expressions into the other. For example, to prove that the expression $ac + ad + bc + bd$ is equivalent to the expression $(a + b)(c + d)$, one could begin by applying the distributive property to the first two terms of the first expression, resulting in $a(c + d) + bc + bd$. Next, one could apply the distributive property to the last two terms, resulting in $a(c + d) + b(c + d)$. Finally, one could apply the distributive property to the two addends $a(c + d)$ and $b(c + d)$, resulting in $(a + b)(c + d)$. Because the expression $ac + ad + bc + bd$ can be transformed into $(a + b)(c + d)$ using properties of operations, the two expressions must name the same number regardless of which value is substituted into them. (If any exception existed, then that exception would also be an exception to the properties of operations, but the properties of operations are true for all numbers without exception.)

In task 7:1, the distributive property can be used to transform the expression $P + 0.0625*P$ into the expression $P*1.0625$. This shows that the two expressions will always give the same result for the total cost, regardless of the before-tax price. This is just one example of how algebra allows us to make statements or draw conclusions that refer to infinitely many cases.

An expression records operations with numbers and with letters standing for numbers. The expression in part (1) records the calculation "Multiply P by 0.0625 then add the result to P." The expression in part (2) records the calculation "Multiply P by 1.0625." The fact that one expression involves addition while the other involves multiplication is a hint that the distributive property is involved. This is because the mathematical relationship between multiplication and addition is given by the distributive property.[†]

The observation that multiplying P by 0.0625 then adding the result to P is equivalent to multiplying P by 1.0625 foreshadows the mathematics of exponential functions. In high school mathematics courses, students take advantage of the equivalence between $P + rP$ and $(1 + r)P$ to create exponential functions. For example, in compound interest we might consider that after the first interest period, an amount of principal P has increased in value to $P + rP = (1 + r)P$; after the second interest period, the principal has increased in value to

$$\begin{aligned} &(1 + r)P + r((1+ r)P) \\ &= (1 + r)((1 + r)P) \\ &= (1 + r)^2P, \end{aligned}$$

and so on, leading to consideration of the expression $(1 + r)^nP$ which defines an exponential function of the variable n.

> 7:1 The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

Answer

(1) In dollars, the cost is 264 + 0.0625*264 = 264 + 16.50 = 280.50. **(2)** In dollars, the cost is 264*1.0625 = 280.50. **(3)** Applying the distributive property, $P + 0.0625*P = 1*P + 0.0625*P = (1 + 0.0625)*P = 1.0625*P$.

Click here for a student-facing version of the task.

Refer to the Standards

7.RP.A.3, 7.EE.A; MP.2, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency, Application

Additional notes on the design of the task

- By convention, multiplication symbols are frequently omitted in algebraic expressions, but multiplication symbols are not omitted here, so as to make the role of multiplication in the task more explicit. The symbol is the one often used in technology.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: calculating tax by finding a percent of a total; and applying properties of operations to rewrite expressions in equivalent forms, especially in cases where a variable written without a coefficient must be recognized as having a coefficient of 1.

Extending the task

How might students drive the conversation further?

- Students could ask or be asked to substitute several different costs into the expressions *P* + 0.0625**P* and *P**1.0625 to verify that the expressions give the same result in different cases. Students could discuss whether the two expressions will give the same result for every possible value of the cost. If students think so, what could be a mathematical justification for that belief? For example, students could be asked to consider the expression *P* + 0.0625**P* - *P**1.0625. What does the expression simplify to?
- Students could discuss which of the two expressions is more convenient for calculating the total cost. This is an example of the way rewriting expressions in equivalent forms can serve a practical or mathematical purpose.

Related Math Milestones tasks

7:3

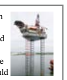
7:3 Write each sum as a product with the given factor. *Example:* $8 + 6x = 2 \cdot ?$ *Answer:* $8 + 6x = 2(4 + 3x)$. **(1)** $6y + 12 = 3 \cdot ?$ **(2)** $-5w + 35 = (-5) \cdot ?$ **(3)** $4z + 1 = 4 \cdot ?$ **(4)** $9ay - 9by + 27cy = (9y) \cdot ?$

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$. **(2)** Show calculation 1(a) on a number line.

7:8

7:8 In 2018, an oil company rented an oil rig for \$100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for \$70 in 2018. **(2)** The company estimates that the profit, *P*, in millions of dollars, after pumping oil for *D* days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make \$30M of profit? **(3)** An equivalent expression for *P* is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?



8:2

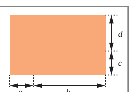
8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

6:2

6:2 **(1)** Would you prefer 33% of a \$100 prize or 75% of a \$50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a \$16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a \$100 item adds ___ dollars to the cost. A 3% tax on a \$1 item adds ___ dollars to the cost.

6:11

6:11 The diagram shows a rectangle. The variables *a*, *b*, *c*, and *d* are lengths in meters.



(1) Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

Tasks **7:3 Writing Sums as Products**, **7:9 Calculating with Rational Numbers**, and **7:8 Oil Business** also involve the distributive property.

In later grades, task **8:2 Pottery Factory** involves a situation that can be modeled with an equation in which properties of operations including the distributive property would be used in the process of solving for an unknown value.

In earlier grades, task **6:2 Prizes, Prices, and Percents** involves some of the same quantities as in task 7:1 (in particular, see parts (4) and (5)). In task **6:11 Area Expressions**, students generate different expressions for the same quantity and use properties of operations to transform one of the expressions into one of the others.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:1? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:1? In what specific ways do they differ from 7:1?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Note that many familiar techniques in algebra can be understood as applications of the distributive property; for examples, see the Teacher Note for Task 7:3, Writing Sums as Products.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

7:2 Utility Pole Scale Drawing

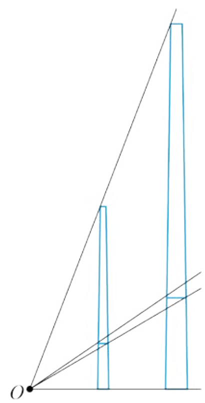
Teacher Notes

Math Milestones™

Central math concepts

To make a scale drawing is to make a mathematical model. A scale drawing of an object is a model of the object that accurately reflects relationships between the object's parts, in the sense that ratios of lengths measured on the drawing are equivalent to ratios of lengths measured directly on the object. For example, when measured in inches directly on the utility pole, the ratio of the two specified diameters is 9:15, and when measured in grid units on the drawing, the ratio of those same two diameters is $\frac{3}{4}$:$\frac{5}{4}$. These two ratios are equivalent; the scale factor that relates them is 12: $12 \times \frac{3}{4} = 9$, and $12 \times \frac{5}{4} = 15$. The scale factor that relates the two ratios is 12 because the measurements involved are respectively in inches and in grid units, and 1 grid unit on the drawing corresponds to 12 inches on the physical object.

Scale drawings are an intuitive application for the idea of "same shape, different size" that informs more precise concepts of geometric similarity. From the grade 8 perspective, shrinking or enlarging a scale drawing could be thought of as the effect of a dilation transformation with $r > 1$ or $r < 1$, respectively (see figure).



Dilating a scale drawing

All models simplify reality, and a scale drawing of a utility pole is a simplification in that a utility pole is three-dimensional, whereas the drawing is flat (two-dimensional). That said, the scale drawing leaves out little information by taking advantage of the rotational symmetry of the pole. There could also be situations, however, in which a three-dimensional scale model would be needed—for example, to test the effects of high winds on a tall utility pole (see figure).

Luo, Y. & Wang, Yucheng & Xie, J. & Yang, C. & Zheng, Yanfeng. (2017). Aero-elastic wind tunnel test of a high lighting pole. Wind and Structures, An International Journal. 25. 1-24.

Relevant prior knowledge

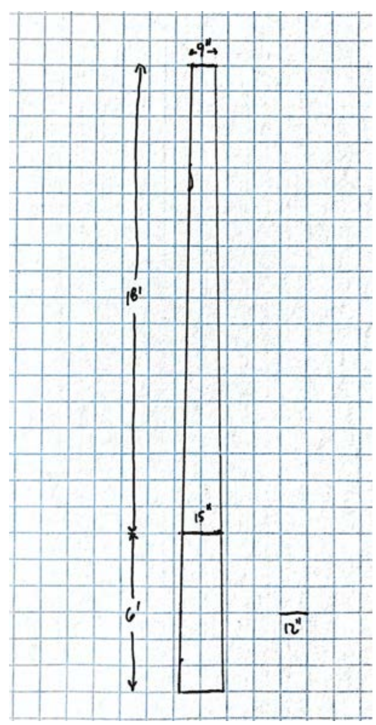
The following mathematics knowledge may be activated, extended, and deepened while students work on the task: converting between feet and inches; fraction reasoning with fourths and eighths; finding equivalent ratios; using scales and units; and applying circle measurement in context.

7:2 A utility pole 24 feet long has $28\frac{1}{4}$-inch circumference at the top and $47\frac{1}{8}$-inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale $\frac{1}{4}$ inch = 1 foot.

Answer

See example. The diameter of the utility pole at the top is 9"; in the scale drawing, the diameter is $\frac{3}{4}$ of a grid unit long. The diameter of the utility pole 6 feet from the base is 15"; in the scale drawing, this is one and a quarter ($\frac{5}{4}$) grid units.

Note: It is not required to determine the diameter of the pole at the base. The diameter at the top and the diameter 6 feet from the base can be drawn, and then the sides of the pole can be drawn starting from the ends of the top diameter, passing through the ends of the diameter 6 feet from the base, and extending to the full length of the pole.



Click here for a student-facing version of the task.

Extending the task

How might students drive the conversation further?

- Students could wonder whether (or be asked to investigate whether) the ratio

  (diameter at the top):(diameter 6 feet from the base)

  is equivalent to the ratio

  (circumference at the top):(circumference 6 feet from the base).

  How could this equivalence be explained algebraically using the formula for the circumference of a circle?
- The scale drawing could be produced using technology, as in [this example](#).

Related Math Milestones tasks

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

8:11

8:11 Study a proof of the Angle-Angle criterion for triangle similarity. Explain one step of the proof in your own words.

6:10

6:10 In the month of February 2021, there were 20 weekdays and 8 weekend days. Here are some questions about that month. **(1)** (Circle all of the correct answers.) The ratio of weekdays to weekend days was 20:8 10:4 5:2 5:7. **(2)** There were ___ times as many weekdays as weekend days. **(3)** True or false: $\frac{5}{7}$ of the days that month were weekdays. **(4)** Approximately what percent of the days that month were weekdays?

Task **7:13 Wire Circle** involves circle measurement in the context of finding a value that satisfies a constraint.

In later grades, task **8:11 Angle-Angle Similarity Proof** involves concepts of geometric similarity that are implicit in task 7:2.

In earlier grades, task **6:10 Weekdays and Weekend Days** involves equivalent ratios.

Refer to the Standards

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Aspect(s) of rigor:

Application

Additional notes on the design of the task

- The scale $\frac{1}{4}''$ = 1 foot may be convenient when using graph paper with quarter-inch squares. If desired, the task could be modified by specifying a different scale, such as $\frac{1}{5}''$ = 6″, which may be convenient when using graph paper with $\frac{1}{5}''$ squares.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:2? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 7:2? In what specific ways do they differ from 7:2?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:2 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

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7:2 Utility Pole Scale Drawing

Teacher Notes

Math Milestones™

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Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

7:3 Writing Sums as Products

Teacher Notes

Central math concepts

The mathematical relationship between multiplication and addition is given by the distributive property:

$$a(b + c) = ab + ac.$$

The distributive property is an *identity*, an equation that is true for all possible values of its variables. The left-hand side of the identity is a product of two terms (one of which is a sum), while the right-hand side of the identity is a sum of two terms (both of which are products). The distributive property allows us to rewrite sums as products and rewrite products as sums.

Many familiar techniques in algebra can be understood as applications of the distributive property:

- Distributing. When we distribute r in the example $r(k + p + 2) = rk + r(p + 2)$, we are applying the distributive property to rewrite a product of two terms as a sum of two terms. Note that we could apply the distributive property again to rewrite $r(p + 2)$ as $rp + 2r$. That would result in the identity $r(k + p + 2) = rk + rp + 2r$.
- Factoring. When we factor out y in the example $xy + yz = y(x + z)$, we are applying the distributive property to rewrite a sum of two terms as a product of two terms.
- Collecting like terms. When we collect like terms in the example $\frac{1}{8}x + -\frac{1}{2}x + \frac{1}{4}x = (\frac{1}{8} + -\frac{1}{2} + \frac{1}{4})x$, we are applying the distributive property to rewrite a sum of three terms as a product of two terms. Of course we could continue to evaluate the sum $\frac{1}{8} + -\frac{1}{2} + \frac{1}{4} = -\frac{1}{8}$, which would result in the identity $\frac{1}{8}x + -\frac{1}{2}x + \frac{1}{4}x = -\frac{1}{8}x$.

Sometimes the distributive property is described as saying that "multiplication distributes over addition." One could equally well say that multiplication distributes over subtraction: $a(b - c) = ab - ac$. This is because subtracting c is the same as adding $-c$, so the distributive property applies as usual. (In more detail, the difference $b - c$ could be rewritten as a sum, $b + -c$, and then $a(b + -c) = ab + a(-c)$ by the distributive property. And because $a(-c) = -ac$, we have $a(b + -c) = ab + -ac$, which we could rewrite finally as $a(b - c) = ab - ac$.)

Similarly, division distributes over addition and subtraction, in the sense that $(a \pm b) \div c = a \div c \pm b \div c$, where $c \neq 0$. This is because dividing by c is the same as multiplying by $\frac{1}{c}$, so the distributive property applies as usual. (In more detail, the quotient $(a \pm b) \div c$ could be rewritten as a product, $(a \pm b) \times \frac{1}{c}$, and then $(a \pm b) \times \frac{1}{c} = a \times \frac{1}{c} \pm b \times \frac{1}{c}$ by the distributive property. We could rewrite this finally as $(a \pm b) \div c = a \div c \pm b \div c$.)

Note in the above our implicit assumption that every c has an additive inverse $-c$, and every nonzero c has a multiplicative inverse $\frac{1}{c}$. Thus,

7:3 Write each sum as a product with the given factor. *Example:* $8 + 6x = 2 \cdot ?$ *Answer:* $8 + 6x = 2(4 + 3x)$. **(1)** $6y + 12 = 3 \cdot ?$ **(2)** $-5w + 35 = (-5) \cdot ?$ **(3)** $4z + 1 = 4 \cdot ?$ **(4)** $9ay - 9by + 27cy = (9y) \cdot ?$

Answer

(1) $6y + 12 = 3\cdot(2y + 4)$. **(2)** $-5w + 35 = (-5)\cdot(w - 7)$. **(3)** $4z + 1 = 4\cdot(z + \frac{1}{4})$. **(4)** $9ay - 9by + 27cy = 9y(a - b + 3c)$. Equivalent expressions are also correct, for example, $-5w + 35 = (-5)\cdot(w + -7)$ for part (2), $4z + 1 = 4\cdot(1z + 0.25)$ for part (3), etc.

Click here for a student-facing version of the task.

Refer to the Standards

7.EE.A.1; MP.1, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Additional notes on the design of the task

As indicated by the example, the desired answer for each part of the task is an equation that shows the equivalence between the initial form of the expression and the factored form. So if a student answers the first part of the task with "? = $2y + 4$," that isn't wrong at all, but in that case, prompt for a fuller answer in the form $6y + 12 = 3(2y + 4)$.

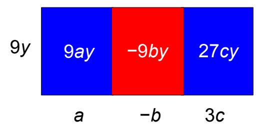
we were relying not only on the distributive property but also on other properties of operations as well. The properties work together to form a system, a system in which calculations can be made and expressions can be transformed.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: understanding of division as an unknown factor problem; the distributive property; rules for operating with signed rational numbers; and viewing expressions as objects with structure.

Extending the task

How might students drive the conversation further?



- Students could draw an area diagram to represent one of the problems (see figure).
- Students could ask or be asked if the unknown factors could be found by division. For example, if $6y + 12$ equals three times an unknown factor, then the unknown factor is $6y + 12$ divided by 3 (or equivalently $\frac{1}{3}$ times $6y + 12$).
- Students could choose values for the variables and verify that the left and right hand sides of the equations are equal when values are substituted.

Related Math Milestones tasks

7:1

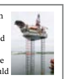
7:1 The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$. **(2)** Show calculation 1(a) on a number line.

7:8

7:8 In 2018, an oil company rented an oil rig for \$100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for \$70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make \$30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

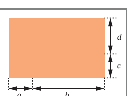


8:2

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

6:11

6:11 The diagram shows a rectangle. The variables a, b, c, and d are lengths in meters.



(1) Using the variables, write three different expressions for the area of the rectangle. **(2)** Choose two of your expressions and show that they are equivalent by applying properties of operations. **(3)** State the property or properties you used.

5:1

5:1 A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 *six-packs* of juice boxes. How many extra juice boxes did the school buy?

Tasks **7:1 Phone Cost**, **7:9 Calculating with Rational Numbers**, and **7:8 Oil Business** also involve the distributive property.

In later grades, task **8:2 Pottery Factory** involves a situation that can be modeled with an equation in which properties of operations including the distributive property would be used in the process of solving for an unknown value.

In earlier grades, task **6:11 Area Expressions** involves transforming expressions into equivalent forms using properties of operations, and task **5:1 Juice Box Mixup** has an interpretation in terms of multiplication distributing over subtraction.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:3? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:3? In what specific ways do they differ from 7:3?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:3 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

7:3 Writing Sums as Products

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

7:4 “Foul Play”

Teacher Notes

Central math concepts

"Will I get rained on today?" and “Will I get struck by a meteorite today?” are two questions that could both be answered “It’s possible,” but thanks to probability we don’t have to leave it at that. Probability allows us to rate the likelihood of two events on a 0–1 scale, allowing us to make quantitative comparisons using ratios and multiplicative comparison. For example, we might estimate that getting struck by a meteorite today is hundreds of times, or thousands of times, or millions of times less likely than getting rained on today. In this way probability makes it possible to be quantitative about likelihood, belief, and risk, giving us a powerful tool for coping with uncertainty.

In games and sports, strategy and decision making can be the difference between winning and losing. In task 7:4, a defender made a split-second decision and later wondered if it was best. Simulation is a good way to analyze such questions. For example, to simulate three successive free throws, a student could randomly generate three whole numbers between 1 and 10 using concrete objects or technology. (As one technology option, the figure shows a Google die rolling feature with a ten-sided die option; a non-pictorial version is here).

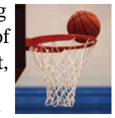
Ahead of time, designate a particular number, such as 4, to represent a missed free throw. The reason one in ten numbers is designated as a “miss” is that the 9 in 10 chance the Pistons player makes a free throw corresponds to a 1 in 10 chance the Pistons player misses a free throw.

Example: Designate 4 as representing a missed free throw and suppose the random numbers generated are 7, 4, and 3. Then this sequence represents the compound event of “make the first free throw, miss the second, and make the third.” That sequence doesn’t win the game for the Pistons as time is running out, because the score had been 100-98, so the Pistons player must make all three free throws to win the game as time is running out.

A dozen students or a dozen pairs of students could generate a sequence of three random numbers, with each student/pair recording whether or not the sequence corresponds to a win for the Pistons as time is running out. The table collects the results from a dozen hypothetical students/pairs.

Student/pair	Random number sequence	Interpretation	Pistons win as time is running out?
A	7, **4**, 3	make, **miss**, make	No
B	7, 10, 9	make, make, make	Yes
C	3, 5, 6	make, make, make	Yes

7:4 *“Foul Play.”* The Hawks were leading the Pistons in basketball by a score of 100-98. Just as time was running out, a Pistons player tried a 3-point shot. His defender had two choices: allow the shot, or stop it by fouling the Pistons player. Fouling would give the Pistons player 3 one-point free throws. The defender chose to foul and later wondered if it was a good choice. **(1)** To analyze the defender’s choice, let’s assume that for the Pistons player, every 3-point shot has probability $\frac{1}{3}$ of going in, and every free throw has probability 90% of going in. **(a)** If the defender allows the shot, what is the probability that the shot wins the game as time runs out? **(b)** If the defender stops the shot by fouling, estimate the probability that the free throws win the game. **(2)** Write a paragraph arguing for or against the defender’s choice, based on probability calculations and/or simulations.



Answer

(1) (a) An exact answer of $\frac{1}{3}$, $0.\overline{3}$, or $33.\overline{3}$,%, or a truncated base-ten form such as 0.333, 33%, etc. **(b)** The probability is about 73%; any estimate from about 53% to about 93% should be considered accurate, since these values are all significantly greater than 33%. **(2)** Responses may vary but should at minimum acknowledge that, given the assumptions of the model, the probability of the Pistons player making 3 free throws in a row is significantly greater than the probability of the Pistons player making the 3-point shot. Answers could also include an evaluation of the reasonableness of the assumptions of the model, and/or a second analysis based on assumptions felt to be more reasonable.

Note: It is possible that the Pistons player could shoot free throws and make exactly two, sending the game to overtime—but the task doesn’t require students to address that possibility in their response. Students could still choose to address that possibility however, either qualitatively or quantitatively.

Click here for a student-facing version of the task.

Student/pair	Random number sequence	Interpretation	Pistons win as time is running out?
D	7, 9, 3	make, make, make	Yes
E	6, 10, **4**	make, make, **miss**	No
F	9, 3, 8	make, make, make	Yes
G	9, 5, 8	make, make, make	Yes
H	9, 1, 6	make, make, make	Yes
I	3, **4**, 1	make, **miss**, make	No
J	7, 9, 2	make, make, make	Yes
K	5, 2, 8	make, make, make	Yes
L	**4**, 1, 2	**miss**, make, make	No

In this simulation, it happened 8 times that the Pistons won on free throws as time was running out, and it happened 4 times that they did not. Based on this simulation, an estimate of the probability that the free throws win the game as time is running out is $\frac{8}{12}$ or about 67%.† The estimate of 67% is a reasonable answer for part (1b) of the task.

It can be tempting to see meaningless patterns in random numbers, such as by observing that the table contains many more odd numbers than even numbers. This is one form of common probability misconception (see "Clustering Illusion" and related topics). Reflective experiences with chance processes can build intuition about signal and noise, and can surface widespread misconceptions about chance and variability that interfere with sound decision making.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: converting between fractions, decimals, and percents and using number sense of fractions, decimals, and percents; comparing numbers multiplicatively; and using technology.

Extending the task

How might students drive the conversation further?

- Students could replace the probabilities $\frac{1}{3}$ and 90% with other values, to investigate the situation for a more accurate three-point shooter or a less accurate free throw shooter. For example, a student knowledgeable about NBA basketball might observe that most players who shoot 90% from the free throw line tend to shoot significantly better than 33% from the three-point line. Note that a simulation with more typical values of these numbers might require more simulation data to accurately estimate the probabilities in the task.
- Students might discuss what a good strategy could be if the Pistons player were very accurate at both three-point shots *and* free throws.

Refer to the Standards

7.SP.C; MP.4, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- Students may not have knowledge of basketball rules, and/or students may not be familiar with typical game situations. Knowledge of rules could be shared, and the task could be acted out to illustrate the situation. Another benefit of acting out the situation is that it could remove the need for students to read the text of the problem. Key information could be elicited and summarized as bullet points.
- The task includes an explicit modeling assumption that "for the Pistons player, ... every free throw has probability 90% of going in." The stated assumption is a strong one; for example, the model excludes consideration of whether the Pistons player will get nervous on the free throw line.
- The defender *later* wondered if fouling had been a good choice. The task doesn't suggest that a real basketball player would, or should, perform a quantitative probability analysis in the split-second game situation.

Related Math Milestones tasks

7:11

7:11 Nechama is shopping online for a ticket to a play. Website A offers a discount of $7.50 off the theater price. Website B offers a discount of 25% off the theater price. **(1)** Is it mathematically possible that Website A is a better deal than Website B? **(2)** Is it mathematically possible that Website B is a better deal than Website A? *Prove your answers.*

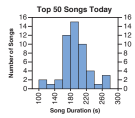
PALACE THEATER ADMIT ONE

6:10

6:10 In the month of February 2021, there were 20 weekdays and 8 weekend days. Here are some questions about that month. **(1)** (Circle all of the correct answers.) The ratio of weekdays to weekend days was 20:8 10:4 5:2 5:7. **(2)** There were ___ times as many weekdays as weekend days. **(3)** True or false: $\frac{5}{7}$ of the days that month were weekdays. **(4)** Approximately what percent of the days that month were weekdays?

6:7

6:7 **(1)** Look up the 50 top songs on a music streaming service. Type each song's duration into a spreadsheet. **(2)** Write a sentence about the data giving a measure of center and a measure of variability. **(3)** Make a histogram of the data.* **(4)** Write a sentence describing the overall pattern of the distribution and any striking deviations from the overall pattern. **(5)** Imagine that one year from now, you go back online and repeat (1)–(4). In what ways would you expect the data distribution to look similar? What differences would you expect to see?



*Use this histogram for (4) and (5) if you don't do (3).

Task **7:11 Ticket Offers** involves decimals and percents as well as the role of the quantitative in decision making.

In later grades, high school students will develop techniques in probability such as multiplying the probabilities of independent events to calculate the probability of a compound event.

In earlier grades, task **6:10 Weekdays and Weekend Days** relates ratios to fractions and percents. Task **6:7 Song Length Distribution** involves a data distribution that arises from sampling variability, as compared to the chance variability which is the issue in task 7:4.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:4? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:4? In what specific ways do they differ from 7:4?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:4 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† The table was produced using a random number generator. Note that with twelve or more students/pairs, it is very likely that the simulation will result in an estimate in the range from about 53% to 93%. A greater number of three-number sequences could be generated to improve the estimate.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students’ interests, experiences, or funds of knowledge?
- How can you build students’ self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students’ agency to the point where they notice and make these choices to solve problems?
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7:5 Is There a Solution? (Addition)

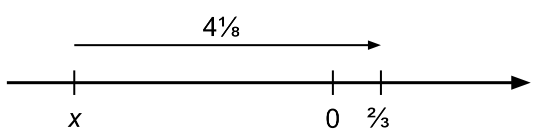
Teacher Notes

Central math concepts

An equation can be viewed as a question: Which values from a specified set, if any, make the equation true? Solving an equation is a process of reasoning resulting in a complete answer to that question.

Task 7:5 also explores the extension of the number system from the nonnegative numbers to the rational numbers. Using only whole numbers, the equation $x + 1 = 0$ cannot be solved; but in the rational number system, any equation of the form $x + a = b$ can be solved. A key reason for this is that for any rational number a, there exists a rational number $-a$, called the additive inverse of a, for which $-a + a = 0$. In terms of this additive inverse, the solution to the equation $x + a = b$ is given by $x = b + -a$. This can be verified by substitution: the value $x = b + -a$ makes the equation $x + a = b$ true because $(b + -a) + a = b + (-a + a) = b + 0 = b$.

The questions in task 7:5 could be settled procedurally by subtracting $4\frac{1}{8}$ from both sides of the equation and then calculating the value of the difference $\frac{2}{3} - 4\frac{1}{8} = -\frac{83}{24}$. However, the task does not ask for the exact value of x. The task thereby aims at non-procedural skills, especially the algebraic skill of looking for and making use of structure (CCSS MP.7), which grows in importance throughout students' study of algebra. And the task prioritizes non-procedural knowledge, such as understanding addition as an operation of translation left or right along the number line, as illustrated in the figure. The figure shows that if a translation by $4\frac{1}{8}$ units from starting point x ends at the point $\frac{2}{3}$ on the number line, then x must be located to the left of 0 on the number line.



Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: addition and subtraction concepts for rational numbers; number sense of the size of fractions; representing problems on a number line; understanding subtraction as an unknown addend problem; and solving one-step equations.

Extending the task

How might students drive the conversation further?

- Students could ask or be asked whether it is mathematically possible to write an equation of the form $x + a = b$ that has no solution even among the rational numbers. As part of this discussion, students could consider

7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

Answer

There is no positive number that solves the equation. There is a negative number that solves the equation. (Reasoning for these decisions may vary.)

Click here for a student-facing version of the task.

Refer to the Standards

7.NS.A.1, 7.EE.B.4; MP.1, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- The intent of saying "pencils down" is to invite a conceptual approach.
- Similarly, the intent of the particular numbers chosen ($4\frac{1}{8}$ and $\frac{2}{3}$) is to invite a conceptual approach.

what happens when -a is added to both sides of the equation. (Recall from the properties of operations that for every number a, there exists a number -a such that -a + a = 0. Recall too from the properties of operations that q + 0 = q for every number q.)

- Students could ask or be asked whether it is mathematically possible to write an equation of the form $x + a = b$ that has two different solutions.
- Students could make sense of the equation $x + 4\frac{1}{8} = \frac{2}{3}$ by creating word problems in which the answer is the solution to the equation. (For example, "The temperature outside increased by $4\frac{1}{8}$ degrees. After that, the temperature was $\frac{2}{3}$ of a degree above zero. What was the temperature before the temperature increased?")
- Students could repeat task 7:5 for an equation like $x + 4\frac{1}{8} = -2$ in which the right-hand side is negative. Without solving the equation, try to decide whether the solution to this equation is greater than or less than the solution to the original equation $x + 4\frac{1}{8} = \frac{2}{3}$.

Related Math Milestones tasks

7:9

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1-1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$. **(2)** Show calculation 1(a) on a number line.

7:12

7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

6:5

6:5 **(1)** Which of the numbers $5, -7, \frac{2}{3}, -\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$. **(3)** Explain why $-(-0.2) = 0.2$ makes sense.

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

Task **7:9 Calculating with Rational Numbers** includes calculations that exercise conceptual understanding of addition and subtraction of rational numbers. Task **7:12 Temperature Change** is a word problem that involves calculating with rational numbers.

In later grades, task **8:9 Water Evaporation Model** involves a linear function with a negative rate of change, putting rational number arithmetic to use for modeling a situation.

In earlier grades, task **6:5 Positive and Negative Numbers** involves concepts of absolute value and order for rational numbers on the number line. Task **6:13 Is There a Solution? (Multiplication)** involves the extension of the number system from whole numbers to fractions.

Curriculum connection

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2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:5 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

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7:6 Car A and Car B

Teacher Notes

Central math concepts

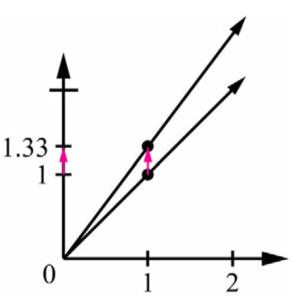
This task centers on the connections between:

- The unit rate in a proportional relationship,
- The point $(1, r)$ on the graph of a proportional relationship, and
- The slope of the graph of a proportional relationship.

To appreciate these connections, first recall that the equation of a proportional relationship between two variable quantities x and y can be written as $y = rx$, where the constant r is the unit rate. This equation says that as x and y vary together, they vary in such a way that y is always r times as much as x. In particular, if $x = 1$, then we will have $y = r \cdot 1$, or simply $y = r$. This implies that the graph of a proportional relationship includes the point $(1, r)$ where r is the unit rate. (We are assuming that 1 is a possible x-value in the given context.) Another particular case is when $x = 0$, assuming that 0 is a possible x-value in the given context; and if $x = 0$, then we will have $y = r \cdot 0$, or simply $y = 0$. This implies that the graph of a proportional relationship includes the point $(0, 0)$.

Therefore we could use the two points $(0, 0)$ and $(1, r)$ to calculate the slope of the graph. The result of that calculation is $\frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(r - 0)}{(1 - 0)} = \frac{r}{1} = r$.

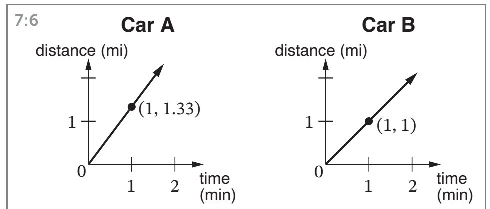
The conclusion we can draw from this calculation is an important one: the slope of the graph of a proportional relationship is the unit rate.



Notice that because the points $(1, 1.33)$ and $(1, 1)$ have the same x-coordinate, the point with the larger y-coordinate is necessarily located higher in the coordinate plane, and this results in that graph being the steeper of the two. This may be easier to see if the two graphs are overlaid on one another, as in the figure. This observation connects the fact that the steepness of the graph is measured by the slope with the fact that the slope is the unit rate.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: interpreting a coordinate pair in the context of a situation by referring to the two coordinate axes and the quantities they refer to in the situation; recognizing and interpreting the unit rate in a proportional relationship; relating the unit rate to an appropriate statement in ratio language (such as, "For every 3 minutes that pass, Car A travels approximately 4 miles"); and writing a two-variable equation to represent a proportional relationship.



Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

Answer

(1) At the end of the first minute, Car A had gone 1.33 miles, and Car B had gone 1 mile. **(2)** Car A was moving faster than Car B. **(3)** $d = 1.33n$ (if d stands for the number of miles moved; different letters could be used for this quantity). **(4)** Using the formula, the car moves $d = 1.33(10) = 13.3$ miles in 10 minutes.

Click here for a student-facing version of the task.

Refer to the Standards

7.RP.A.2; MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

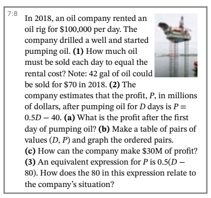
Extending the task

How might students drive the conversation further?

- Students could be asked to sketch a graph for a third car ("Car C") that is specified to be moving at a constant speed of 0.27 miles per minute. A quantitative approach to sketching the graph might be to first estimate the location of the point (1, 0.27) and draw the graph as a straight line from the origin through that point.
- Students could use a spreadsheet to enter the formula they created in part (3) and use the formula to quickly create a two-column table of *x*-*y* ordered pairs and graph the pairs of values in the table. This could be done for the slower car, as well, allowing the two graphs to be shown on the same plot.

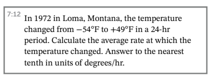
Related Math Milestones tasks

7:8



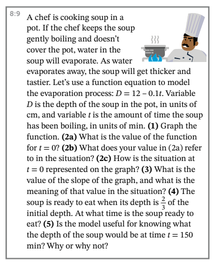
7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

7:12



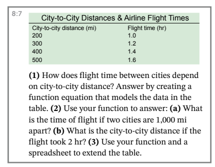
7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

8:9



8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

8:7



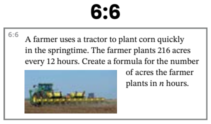
8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

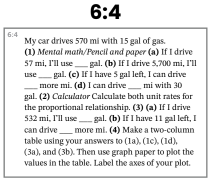
(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

6:6



6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:4



6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Another task for grade 7 that prominently features unit rates is **7:8 Oil Business**. Task **7:12 Temperature Change** involves an average rate.

In later grades, proportional relationships are extended to linear functions. Tasks **8:9 Water Evaporation Model** and **8:7 Flight Times and Distances** prominently feature linear functions.

In earlier grades, students work with unit rates and proportional relationships. Tasks **6:6 Planting Corn** and **6:4 Gas Mileage** prominently feature proportional relationships.

Additional notes on the design of the task

- It may be productive to ask students to share with one another the reasoning that led them to choose Car A or Car B for part (2) of the task. Some students might have chosen Car A based on a visual sense of the greater steepness of its graph; other students might have reasoned numerically: a car that travels 1.33 miles in 1 minute is necessarily moving faster than a car that travels only 1 mile in 1 minute. A mathematical discussion could seek to connect those two approaches.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:6? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 7:6? In what specific ways do they differ from 7:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

7:6 Car A and Car B

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

7:7 Speed Limit

Teacher Notes

Central math concepts

If we divide a measurement with units of length by a second measurement with units of time, the resulting quotient measures a quantity of a third kind, speed. And in a similar way, there are many common situations in which division creates new kinds of quantities out of the quantities being divided. For example, if we pour a quantity of sugar into warm water, then the number of grams of sugar divided by the number of liters of water provides a measure of the sweetness of the solution. If we divide the rise of a line in the coordinate plane by the run, then the resulting quotient provides a measure of the steepness of the line. If we divide the number of persons living in a county by the area of the county in square miles, then the resulting quotient measures the quantity called population density. If we divide the number of kilograms of mass in a substance by the number of cubic meters occupied by the substance, then the resulting quotient measures the quantity called mass density. The number of possible examples is endless.

Measures of speed, sweetness, population density and the like can be compared against one another to say which object is moving faster, which drink is sweeter, which county is more dense, and so on. When two measures of the same kind of quantity are given in the same units, comparing them is straightforward: a car moving at a speed of 65 mph is moving faster than a car moving at a speed of 61 mph, just as a 100-watt light bulb is brighter than a 40-watt light bulb. However, when two measures of the same kind of quantity are given in different units, their numerical values cannot be directly compared. A stick 1 meter long is not the same length as a stick 1 yard long, even though 1 = 1. In this task, we are asked to compare a speed measured in units of kilometers per hour with a speed measured in miles per hour. This invites thinking about whether a kilometer is longer or shorter than a mile, and by what factor. Quantitative literacy includes knowing a reasonable collection of such facts. No two people carry around the same quantitative knowledge of the world in their back pocket, but everyone should carry some.

Yet it is a fact of modern life that when you're connected to the internet with a web browser, you're in immediate contact with a "hive mind" that knows every fact you know and more. The expectation in task 7:7 is that students will have access to technology. What will happen when a student is given task 7:7 and, instead of doing what would have been done forty years ago, the student types "convert 100 km/hr to mph" into a browser search bar and obtains the instantaneous result 62.1371 miles per hour? What mathematical conversations are valuable at that point? Some suggestions are in "**Extending the task**."

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: converting units; working with unit rates; and using appropriate precision in applications.

7:7	If the speed limit in Canada is 100 km/hr and you are driving 65 mph, are you over or under the limit? By how much?

Answer

If using km/hr: You are over the speed limit by about 5 km/hr. If using mph: You are over the speed limit by about 3 mph.

Click here for a student-facing version of the task.

Refer to the Standards

7.RP.A.1; MP.5, MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Additional notes on the design of the task

- The phrasing of the task is mathematically imprecise in the way that everyday language is mathematically imprecise. This leads to the possibility of two different answers depending on the choice of units in which to measure the difference. This intentional ambiguity also leads to the possibility of different answers depending on the student's chosen level of precision. Using mathematics in real life is often less constrained, less dictated, than when mathematics is used in school. The design of this task is intended to give students a small taste of that freedom and the potential confusion such choices can give rise to.

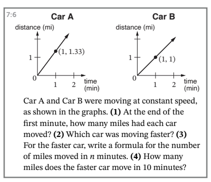
Extending the task

How might students drive the conversation further?

- If a student uses technology to perform a conversion such as 100 km/hr = 62.1371 mph (or if students are shown the output of such a technology-based calculation featuring a similar level of precision), then a worthwhile conversation might be to ask whether all of the digits to the right of the decimal point are useful. How different, in everyday terms, are the two measurements 62.1371 mph and 62.1372 mph? Additional important questions could include, "Is the computer's value correct? How does the computer arrive at its answer?"
- Students could develop a formula that returns the speed in miles per hour given the speed in kilometers per hour. They could also develop a formula that returns the speed in kilometers per hour given the speed in miles per hour.

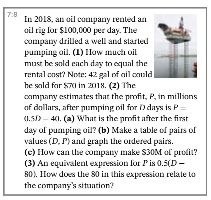
Related Math Milestones tasks

7:6

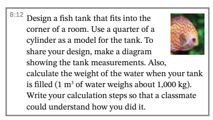

7:6

Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

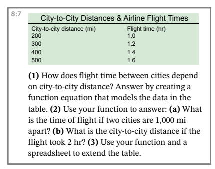
7:8


7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

8:12


8:12 Design a fish tank that fits into the corner of a room. Use a quarter of a cylinder as a model for the tank. To share your design, make a diagram showing the tank measurements. Also, calculate the weight of the water when your tank is filled (1 m^3 of water weighs about 1,000 kg). Write your calculation steps so that a classmate could understand how you did it.

8:7

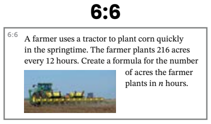

8:7

City-to-City Distances & Airline Flight Times

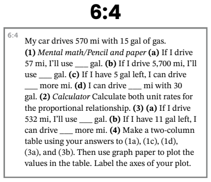
City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

6:6


6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:4


6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Task **7:6 Car A and Car B** prominently features a proportional relationship in which the slope of a line is a visual measure of the speed of motion. Task **7:8 Oil Business** involves unit rates in an application context.

In later grades, task **8:12 Fish Tank Design** involves working with units and compound units. Task **8:7 Flight Times and Distances** prominently features a linear function with a rate of change formed by dividing a quantity of length by a quantity of time.

In earlier grades, tasks **6:6 Planting Corn** and **6:4 Gas Mileage** prominently feature unit rates and proportional relationships.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:7? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:7? In what specific ways do they differ from 7:7?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:7 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

7:8 Oil Business

Teacher Notes

Math Milestones™

Central math concepts

In task 7:8, there are three distinct representations for the relationship between profit *P* and number of days *D*. The three representations are an equation, a table, and a graph. The specific question asked in part (2)(c) of the task, "How can the company make $30M of profit?" could be answered by using any of the three representations - for example, by solving the equation $0.5D - 40 = 30$ to find $D = 140$. This result shows that the company can make $30M of profit by pumping oil for 140 days. As important as it is for students to know how to establish specific facts like this, it is equally important for students to understand how each representation encodes such facts in characteristic ways:

- The graph shows that the company can make $30M of profit by pumping oil for 140 days because the graph includes the point with coordinates (140, 30).
- The table shows that the company can make $30M of profit by pumping oil for 140 days because the table includes a row with 140 in the *D* column and 30 in the *P* column.
- The equation shows that the company can make $30M of profit by pumping oil for 140 days because the result of substituting 140 for *D* in the formula $P = 0.5D - 40$ is $P = 0.5(140) - 40 = 70 - 40 = 30$.

The representations all show the same fact about the situation in different ways. As students gain fluency and confidence with algebra, the economy of the equation representation makes it primary, although graphs and tables remain valuable in many situations in school, work and life.

Representations make mathematical thinking visible, so that the thinking can be discussed, debated, and refined. Representations can also provide access points to the mathematics that rely less heavily on spoken or written language. The value of multiple representations, then, isn't primarily in providing students with multiple methods for getting answers to a given problem; it's in deepening students' grasp of the mathematics from which a given problem emerges.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: interpreting positive and negative numbers in context; substituting values into variable expressions; and using unit rates (unit price and rate per day).

Extending the task

How might students drive the conversation further?

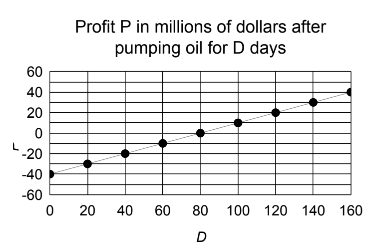
- Students could discuss the two equivalent expressions $0.5D - 40$ and $0.5(D - 80)$ in the task. What quantities in the situation are most visible in each form?

7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, *P*, in millions of dollars, after pumping oil for *D* days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for *P* is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

Answer

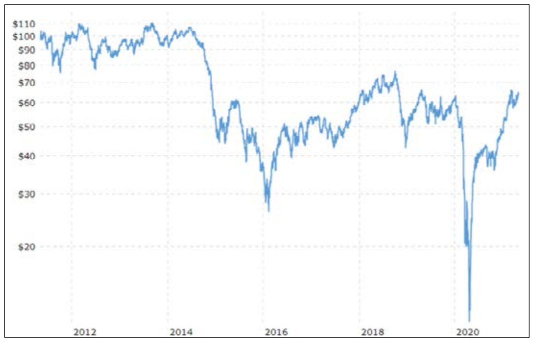
(1) 60,000 gallons per day. Answers that differ from this value because of rounding intermediate steps are acceptable. (For example, if the unit price of oil is rounded to $1.67 per gallon, then to the nearest whole number, 59,880 gallons of oil must be sold.) **(2)** **(a)** -39.5 million dollars. **(b)** Answers may vary; see example from this online spreadsheet. **(c)** Pump oil for 140 days. **(3)** After pumping oil for 80 days, the company's profit is $P = 0.5(80 - 80) = 0$. If the company pumps oil for less than 80 days, then their profit will be negative (meaning that they will lose money). If the company pumps oil for more than 80 days, their profit will be positive.

D	P
0	-40
20	-30
40	-20
60	-10
80	0
100	10
120	20
140	30
160	40



Click here for a student-facing version of the task.

- Students could ask how much the selling price of oil changes during the course of a typical year. A graph like this shows the historical trends. Is oil a risky business?



Related Math Milestones tasks

7:1

7:1 The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

7:14

7:14 Rose and Liba both solved this problem: *Jannat has 4 packs of balloons and 5 single balloons—29 balloons in all. How many balloons are in a pack?* Explain both of Rose's steps. Check that Liba's equations are all true statements about the balloons.

Rose	Liba
$29 - 5 = 24$	Let x be the # of balloons in a pack.
$24 \div 4 = 6$	$4x + 5 = 29$
	$4x = 24$
	$x = 6$

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

8:7

8:7 City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Like task 7:8, task **7:1 Phone Cost** involves a use of the distributive property to transform an expression in context. Task **7:14 Comparing Rose and Liba's Solutions** involves an equation like the one that could be used to answer part (2)(b) of task 7:8.

In later grades, tasks **8:9 Water Evaporation Model** and **8:7 Flight Times and Distances** prominently feature linear function models that are implicit in the company's profit model in task 7:8.

In earlier grades, tasks **6:6 Planting Corn** and **6:4 Gas Mileage** prominently feature unit rates and proportional relationships.

Refer to the Standards

7.RP.A.2b, 7.EE.A.2, 7.EE.B.4; MP.1, MP.2, MP.4, MP.5, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Additional notes on the design of the task

- Compared to the expression $0.5D - 40$, the equivalent form $0.5(D - 80)$ is intended to make it more transparent that 80 days is the dividing line between profit and loss. To verify that $0.5(D - 80)$ is equivalent to $0.5D - 40$, students could apply the distributive property.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:8? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:8? In what specific ways do they differ from 7:8?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:8 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

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Language

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Identity, Agency, and Belonging

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- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

7:9 Calculating with Rational Numbers

Teacher Notes

Central math concepts

The properties of operations allow calculation with positive fractions to be extended to calculation with signed rational numbers.

$$(a + b) + c = a + (b + c)$$

$$a + b = b + a$$

$$a + 0 = 0 + a = a$$

For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.

$$(a \times b) \times c = a \times (b \times c)$$

$$a \times b = b \times a$$

$$a \times 1 = 1 \times a = a$$

For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.

$$a \times (b + c) = a \times b + a \times c$$

On the one hand, many aspects of calculating with rational numbers are already present when calculating with whole numbers or positive fractions. For example, the additive identity property of 0 is used already in kindergarten with whole numbers, and 0 has the same property in the rational number system. As another example, given any positive fraction, there exists a positive fraction (namely the reciprocal) whose product with the given fraction is 1; and reciprocals (multiplicative inverses) exist for all nonzero rational numbers too. Some other aspects of earlier-grades work with number and operations that will continue in the rational number system include:

- Associativity of addition and multiplication.
- The multiplicative identity property of 1.
- Commutativity of addition and multiplication.
- The relationship between addition and subtraction: $C - A$ is the unknown addend in $A + \square = C$.
- The relationship between multiplication and division: $C \div A$ (with A nonzero) is the unknown factor in $A \cdot \square = C$.
- The relationship between multiplication and addition: $A \cdot (B + C) = A \cdot B + A \cdot C$.

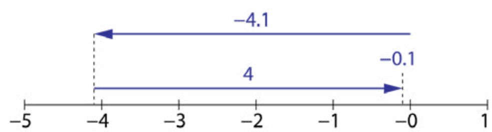
One new aspect of the rational number system is the existence of additive inverses. There is no positive number that solves the equation $1 + x = 0$, but there is one rational number that solves the equation, namely $x = -1$. More generally, every rational number A has a unique additive inverse $-A$ with the defining property $A + -A = 0$.

Also new in the rational number system is the question of how to evaluate products like $-1 \cdot 5$ or $-3 \cdot -4$. In principle, the properties also answer these questions. For example, to evaluate the product $-1 \cdot 5$, one could add 5 to the product and see what happens: $5 + -1 \cdot 5 = 1 \cdot 5 + -1 \cdot 5 = (1 + -1) \cdot 5 = 0 \cdot 5 = 0$. Comparing the first and last expressions, we have $5 + -1 \cdot 5 = 0$, which says that $-1 \cdot 5$ is the additive inverse of 5, or -5. Generalizing such observations justifies procedural guidance such as "negative times positive equals negative."

7:9 **(1)** Calculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $-1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$.
(2) Show calculation 1(a) on a number line.

Answer

(1) (a) -0.1. **(b)** $-\frac{5}{6}$. **(c)** 2. **(d)** $2\frac{1}{2}$ or $\frac{5}{2}$. **(e)** 3. **(f)** $-\frac{1}{3}$. **(g)** 1. **(h)** 0. **(2)** Answers may vary but should reveal or support an explanation of how the result comes about. See the figure for an example.



Click here for a student-facing version of the task.

Refer to the Standards

7.NS.A; MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Additional notes on the design of the task

- The varied calculations in part (1) seek to portray positive and negative fractions and decimals as an integrated system of numbers that can be operated on and transformed.

The prominence of the properties of operations in calculating with rational numbers means that calculating with rational numbers is more like doing algebra than was the case when students calculated with multi-digit whole numbers. There isn't a standard algorithm for evaluating expressions like $-1(-1 - 1)$ or $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$. Instead, there are choices to make, such as whether to evaluate $\frac{1}{2} - \frac{1}{4}$ first or evaluate $-9 + 9$ first. Those choices require comprehension of the structure of expressions, as well as fluency with the syntax of expressions and their conventions (such as omitting the multiplication sign or using the same symbol for subtraction as for negation). In some ways, calculating with rational numbers resembles opportunistic mental calculations from earlier grades, as in problems like 4,999 + 12, in which properties of operations and relationships between operations can be used to rewrite the given expression in a more convenient form, in this case 4,999 + 1 + 11 = 5,000 + 11. Calculation in the elementary grades was never only algorithmic[†], and in the middle grades and high school it seldom ever is.[‡]

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: understanding quotients of whole numbers as fractions; understanding mixed numbers as sums of whole numbers and fractions; multiplying a whole number by a fraction; relating fractions and decimals; using grouping symbols; and looking for structure in expressions.

Extending the task

How might students drive the conversation further?

- Students could generate and compare several possible early steps for a given calculation, such as $(-\frac{3}{8})(-8) = (\frac{3}{8})(8) = (\frac{3}{8})(\frac{8}{1})$ vs. $(-\frac{3}{8})(-8) = (\frac{3}{8})(8) = 3 \cdot \frac{1}{8} \cdot 8$ or other possibilities.
- In discussing errors, students could suggest ways to rewrite the expression so its structure is more transparent, for example:

$$-1(-1 - 1) = (-1)(-1 + -1).$$

Related Math Milestones tasks

7:12

7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

7:3

7:3 Write each sum as a product with the given factor. *Example:* $8 + 6x = 2 \cdot ?$ *Answer:* $8 + 6x = 2(4 + 3x)$. **(1)** $6y + 12 = 3 \cdot ?$ **(2)** $-5w + 35 = (-5) \cdot ?$ **(3)** $4z + 1 = 4 \cdot ?$ **(4)** $9ay - 9by + 27cy = (9y) \cdot ?$

7:5

7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

Task **7:12 Temperature Change** involves arithmetic with signed rational numbers in context, and task **7:3 Writing Sums as Products** involves rewriting algebraic expressions with rational number coefficients. Task **7:5 Is There a Solution? (Addition)** focuses on an equation that has no solution in the positive numbers but that can be solved in the rational number system.

Curriculum connection

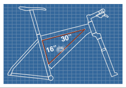
1. In which unit of your curriculum would you expect to find tasks like 7:9? Locate 2–3 similar tasks in that unit. How are the tasks similar to each other, and to 7:9? In what specific ways do they differ from 7:9?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:9 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

8:6

8:6 Write as a fraction in lowest terms: **(1)** $1.041\overline{6}$. **(2)** $3^2 \cdot 3^{-5}$.

8:3

8:3 On this blueprint for building a bike, part of the bike is shaped like a right triangle. The longest side length is illegible because water spilled on the blueprint. Calculate that side length.



In later grades, task **8:6 Rational Form** involves connections between the rational number system and repeating decimals. Task **8:3 Bicycle Blueprint** involves the Pythagorean theorem, the full meaning of which extends the concept of number beyond rational numbers.

6:5

6:5 **(1)** Which of the numbers $5, -7, \frac{2}{3}, -\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$. **(3)** Explain why $-(-0.2) = 0.2$ makes sense.

6:13

6:13 *w* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

6:14

6:14 *Pencil and paper* **(1)** $81.53 \div 3.1 = ?$ **(2)** $\frac{7}{8} \div \frac{2}{3} = ?$ **(3)** Check both of your answers by multiplying.

In earlier grades, task **6:5 Positive and Negative Numbers** concentrates on pre-arithmetic properties of signed rational numbers and their locations on the number line. Task **6:13 Is There a Solution? (Multiplication)** focuses on an equation that has no solution in the whole numbers but that can be solved in the positive fractions. Task **6:14 Dividing Decimals** and Fractions is a procedural task involving quotients of fractions and decimals.

† Adding It Up (2001), p. 121.

‡ William McCallum (2008), "Mindful Manipulation: What Algebra Do Students Need for Calculus?" (presentation)

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

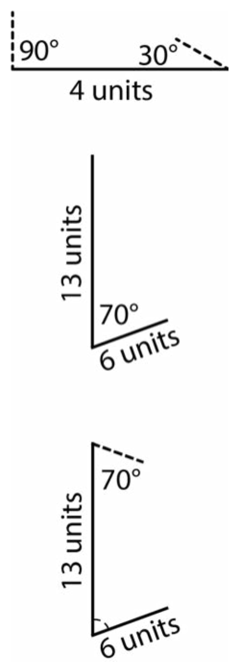
7:10 Triangle Conditions

Teacher Notes

Central math concepts

Imagine that a student comes to you with the following proposal: "Let's each draw a square in the coordinate plane. Both of our squares must have a perimeter of 28 units. You go first." Accepting this curious proposal, you begin by noting to yourself that a square with a perimeter of 28 units necessarily has a side length of 7 units. You draw your square and wait for the student to draw their square. At this point you don't know if the student will draw their square near yours, or if their square will be rotated relative to yours. What you do know, however, is that whatever square they draw, it will be congruent to yours. One could say that knowing the perimeter of a square determines the square "up to congruence."

By contrast, knowing the perimeter of a triangle doesn't determine the triangle up to congruence. For example, if the student's proposal is to draw triangles that must have a perimeter of 240, then you might draw a 15-112-113 right triangle while the student might draw a 40-96-104 right triangle. These triangles aren't congruent.



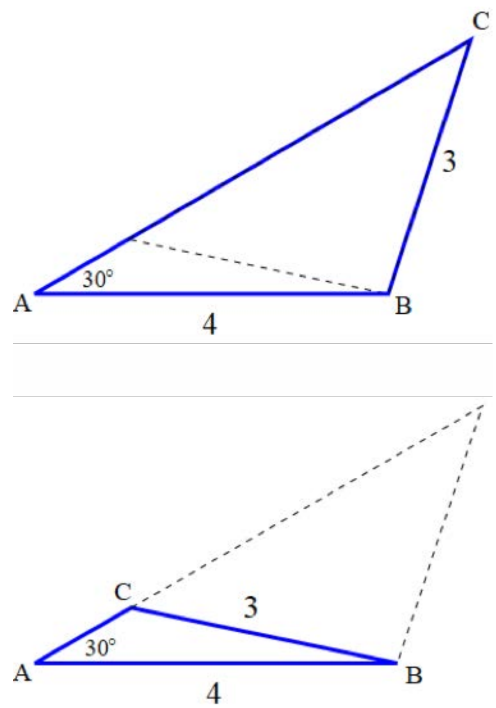
Perimeter doesn't determine a triangle up to congruence, but specifying some kinds of information does determine a triangle up to congruence. Important cases of this include specifying all three side lengths; specifying two angles and the included side (illustrated in the first case shown in the figure); and specifying two sides and the included angle (illustrated in the second case shown in the figure). In high school, students learn to prove these claims deductively, whereas in grade 7 the analysis of such given conditions is not an axiomatic-deductive process. The intuitions and facts that students acquire, however, are useful even in the shorter term, as when the Angle-Angle Similarity criterion for triangles plays a role in the concept of the slope of a line in the coordinate plane in grade 8. The Angle-Angle Similarity criterion, in turn, depends on the Angle-Side-Angle criterion for triangle congruence, an early glimpse of which is visible in the figure, top.

Some conditions for a triangle are impossible to satisfy. This is illustrated in the third case in the figure: there is no triangle with adjacent sides of 6 units and 13 units, and the 70° angle shown. For more discussion of triangle conditions and geometry in grade 7, see the Progression document for 7–8 and High School Geometry† and in the Progression document for 6–8 Expressions and Equations.‡

7:10 In ΔABC, side AB is 4 units long, side BC is 3 units long, and angle A measures 30°. Sketch two ways ΔABC might look.

Answer

See the figure.



Click here for a student-facing version of the task.

Refer to the Standards

7.G.A.2; MP.3, MP.5, MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: using construction tools, including technology; recognizing, measuring, and creating angles; using geometry notation; and constructing mathematical arguments.

Extending the task

How might students drive the conversation further?

- Students could see if there are two different triangles if the measure of angle *A* in the task is specified as 90°.
- Students could see if there are two different triangles if the measure of angle *A* is specified as any positive angle less than 20°.

Related Math Milestones tasks

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

8:11

8:11 Study a proof of the Angle-Angle criterion for triangle similarity. Explain one step of the proof in your own words.

6:12

6:12 **(1)** What is the area of the triangle in the coordinate plane with vertices $(1, 2)$, $(-5, 2)$, and $(-8, 9)$? **(2)** How does the area change if we change the third vertex to $(-3, 9)$?

5:8

5:8 A *scalene triangle* is a triangle in which the sides all have different lengths. Thinking about this, Alana decided there should also be a name for quadrilaterals in which the sides all have different lengths. She said, "I'll name them after myself." She defined an *alana-gon* to be a quadrilateral in which the four sides all have different lengths. **(1)** Draw an example of an alana-gon. **(2)** True or false: **(a)** All squares are alana-gons. **(b)** No trapezoids are alana-gons.

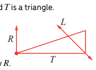
4:8

4:8 L is a line, R is a ray, and T is a triangle. True or false:

(1) Line L is a line of symmetry for triangle T.

(2) Line L intersects ray R.

(3) Triangle T has two angles measuring less than 90 degrees.



An implication of task **7:13 Wire Circle** is that the circumference of a circle is enough information to determine the diameter of the circle (and therefore enough information to determine the circle, up to congruence).

In later grades, task **8:11 Angle-Angle Similarity Proof** focuses on a criterion establishing that if two angle measures are specified in a triangle, then although the triangle isn't specified up to congruence, it is specified up to similarity.

In earlier grades, task **6:12 Coordinate Triangle** specifies a triangle not with length and angle information, but with coordinate values for the vertices. In task **5:8 Alana's Shape Category**, a condition specifies not a single shape but an entire category of non-congruent shapes. Angles and angle measure are involved in task **4:8 Shapes with Given Positions**.

Additional notes on the design of the task

- The task does not include a diagram, because the product the task calls for is a diagram.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:10? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 7:10? In what specific ways do they differ from 7:10?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:10 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 7–8, HS, Geometry*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

‡ Common Core Standards Writing Team. (2011). *Progressions for the Common Core State Standards for Mathematics: 6–8, Expressions and Equations* (Draft, 4/22/2011), p. 12. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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7:10 Triangle Conditions

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

7:11 Ticket Offers

Teacher Notes

Math Milestones™

Central math concepts

Website A and Website B are both subtracting a dollar amount from the theater price, but the dollar amount subtracted by Website A is constant, whereas the dollar amount subtracted by Website B is proportional to the theater price. The smaller the theater price, the smaller the dollar amount subtracted by Website B. Therefore if the theater price is low enough, the dollar amount subtracted by Website B will be less than the dollar amount subtracted by Website A, and Website A will be the better deal. Conversely if the theater price is high enough, the dollar amount subtracted by Website B will be greater than the dollar amount subtracted by Website A, and Website B will be the better deal. The "hidden variable" in task 7:7 is the theater price, which is not given as a number. A breakthrough in this task is to realize that the theater price *is* a variable.

Situations commonly arise that involve a choice between an absolute dollar amount and a percentage. For example, one grocery store might be selling birthday cakes at half-price, while a nearby grocery store might be offering $10 off the price of a birthday cake. Or, when a composer sells a song to a film production company, the composer might have a choice between receiving a fixed dollar amount for the song versus receiving a percentage of the profits generated by the film. Just as the better deal in task 7:11 depends on the theater price of the ticket, the better deal for the cake buyer or the composer depends on estimating the price of a birthday cake or the profit potential of a forthcoming film.

If the quantitative relationships in task 7:11 were expressed algebraically, one could write $A = t - 7.5$ and $B = t - 0.25t$, where t is the theater price in dollars, A is the final cost on Website A in dollars, and B is the final cost on Website B in dollars. Using the distributive property, function B could be rewritten as $B = t(1 - 0.25)$ or, after simplifying, $B = 0.75t$. The price at which the two websites offer the same deal could be found by solving the equation $t - 7.5 = 0.75t$. The function equations $A = t - 7.5$ and $B = 0.75t$ both define linear functions.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: finding percent of a total; mental calculation; and using fraction-decimal-percent equivalents.

Extending the task

How might students drive the conversation further?

- Students might realize that there exists a particular value of the theater price for which Website A and Website B are offering the ticket at the same final cost. (Or students might ask about this, or they could be

7:11 Nechama is shopping online for a ticket to a play. Website A offers a discount of \$7.50 off the theater price. Website B offers a discount of 25% off the theater price. **(1)** Is it mathematically possible that Website A is a better deal than Website B? **(2)** Is it mathematically possible that Website B is a better deal than Website A? *Prove your answers.*

Answer

(1) Yes. Proof: The theater price could be \$10, in which case Website A is offering the ticket at a final cost of \$2.50, which is a better deal than Website B, which is offering the ticket at a final cost of \$7.50. **(2)** Yes. The theater price could be \$100, in which case Website B is offering the ticket at a final cost of \$75, which is a better deal than Website A, which is offering the ticket at a final cost of \$92.50.

Click here for a student-facing version of the task.

Refer to the Standards

7.RP.A.3, 7.EE.B; MP.2, MP.3, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- Theater tickets tend to vary widely in price. Students may want to discuss and agree upon a reasonable range of theater prices to consider, for example \$10 to \$50.

asked about it.) What are some ways of determining this “crossover” value? Consider using tables, graphs, and/or equations.

- Students could compare cases in which giving a single example does, or does not, prove a statement conclusively. For example, suppose the statement to be proved is, “Given any two even whole numbers, their product is even.” Does the example 8 × 6 = 48 prove the statement? Would a hundred specific examples prove the statement? If not, what sort of argument would prove the statement conclusively?

Related Math Milestones tasks

7:1

7:1 The cost of a phone is the phone’s price, $264, plus 6.25% tax. **(1)** Use the expression $P + 0.0625 * P$ to find the cost. **(2)** Use the expression $P * 1.0625$ to find the cost. **(3)** Apply properties of operations to the expression $P + 0.0625 * P$ to produce the expression $P * 1.0625$.

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn’t cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let’s use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

8:7

8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

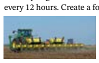
(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

6:2

6:2 **(1)** Would you prefer 33% of a $100 prize or 75% of a $50 prize? **(2)** 8 is 25% of what number? **(3)** 14 is what percent of 200? **(4)** Write 6.25% as a decimal, then as a fraction in lowest terms. **(5)** Find the total cost of a $16 item after a sales tax of 6.25% is added. **(6)** A 3% tax on a $100 item adds ___ dollars to the cost. A 3% tax on a $1 item adds ___ dollars to the cost.

6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.



6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I’ll use ___ gal. **(b)** If I drive 5,700 mi, I’ll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I’ll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Task **7:1 Phone Cost** uses percent in a problem about algebraic expressions and the distributive property.

In later grades, tasks **8:9 Water Evaporation Model** and **8:7 Flight Times and Distances** prominently feature linear functions, which formalize the quantitative relationships underlying task 7:11.

In earlier grades, task **6:2 Prizes, Prices, and Percents** prominently features percent calculations involving dollar amounts. Tasks **6:6 Planting Corn** and **6:4 Gas Mileage** prominently feature unit rates and proportional relationships.

- The phrasing of the task is mathematically imprecise in the way that everyday language is mathematically imprecise. Specifically, the “better deal” between the two websites is intended to refer to the offer with the lower final cost after the discount is subtracted from the theater price. This meaning could be made explicit through discussion or partner work (especially if students aren’t very familiar with purchases, discounts, and deals).

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:11? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:11? In what specific ways do they differ from 7:11?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

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7:11 Ticket Offers

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

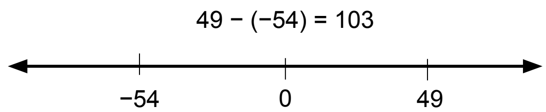
Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

7:12 Temperature Change

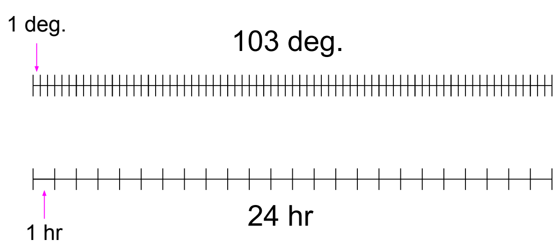
Teacher Notes

Central math concepts



A number line diagram is helpful for representing the initial and final temperatures and for visualizing and calculating the magnitude of the temperature increase. With reference to this number line diagram (not drawn to scale), the initial temperature is at the point marked -54. The first 54 degrees of temperature increase will bring the temperature to zero, then another 49 degrees of temperature increase will bring the temperature to the final temperature of 49 degrees. The total increase is therefore the sum of the two increases, or 54 + 49 = 103 degrees. A procedural version of this calculation could proceed as, "temperature change = final temperature - initial temperature," with the subtraction performed as 49 - (-54) = 49 + 54 = 103.

In the rational number system, subtraction is reducible to addition; this is why there are no properties of operations that refer to subtraction. Specifically, subtraction in the rational number system means "adding the additive inverse," that is, $A - B$ means $A + (-B)$. In particular then, 49 - (-54) = 49 + -(-54), where -(-54) refers to the additive inverse of -54. By definition, the additive inverse of -54 is the number that makes 0 when added to -54, and that number is 54. So -(-54) = 54 and the original subtraction calculation 49 - (-54) becomes 49 + -(-54) = 49 + 54 = 103.



Left out of the above account so far is any sense of how fast the temperature increase happened, or how steady the increase was. Thinking about the average rate of change of a quantity means imagining that the change happened at a constant rate, even if we know that in reality it didn't happen that way. The diagram (not drawn to scale) gives a sense of how the change in temperature of 103 degrees compares to the duration of time over which the change took place, 24 hr. In the diagram, 103 degree units compose the same length as 24 hour units. Because 103 is approximately 4.29 times as much as 24, each degree unit must be smaller than each hour unit by that same factor. In other words, there are about 4.29 degree units for every hour unit. That implies a unit rate of 4.29 degrees per hour for the process, imagining it as a constant-rate process.

Even though the temperature was unlikely to have changed at a constant rate, the average rate provides useful insight of the intensity of the event. The two numbers 103 degrees and 24 hours aren't as informative by themselves as they are when combined with the new fact, produced through division, that in every hour on average, the temperature climbed

> 7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

Answer

4.3 degrees/hr.

Click here for a student-facing version of the task.

Refer to the Standards

7.RP.A.1, 7.NS.A; MP.2, MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- The event described in task 7:12 occurred on January 14th–15th, 1972. See Horvitz et al. (2002), "A National Temperature Record at Loma, Montana."

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:12? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 7:12? In what specific ways do they differ from 7:12?

more than 4 degrees. Perhaps, had we been outside during this event, such a rapid warming trend would have been directly perceptible.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: representing rational numbers on a number line; subtracting signed rational numbers; and understanding unit rates and average rates and relating them to ratios of dimensioned quantities.

Extending the task

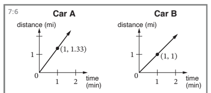
How might students drive the conversation further?

- Students could calculate average rates of temperature change for other extreme weather events, such as the "heat burst" that occurred in Hobart, Oklahoma on May 23, 2005, when temperatures rose from 74.1°F to 93.4°F in a 5-minute period (Christopher C. Burt, "Extreme Short-Duration Temperature Changes in the U.S.").
- Students could plot the given information in the coordinate plane as the pair of points (0, -54) and (24, 49). The average rate of change of temperature corresponds to the steepness of the line joining these two points.

Related Math Milestones tasks

7:6

7:6



Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in *n* minutes. **(4)** How many miles does the faster car move in 10 minutes?

7:8

7:8

In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

8:1

8:1

Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. Assuming a linear model, about how many more hours will it take him to finish summarizing?

6:4

6:4

My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

6:3

6:3

The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

Time	Hours after Midnight	Temp °F
8:00 pm	–4	–42
9:00 pm	–3	–42
10:00 pm	–2	–41
11:00 pm	–1	–40
Midnight	0	–39
1:00 am	1	–39
2:00 am	2	–38

Plot the data on graph paper and label the plot. Describe any patterns you see.

6:5

6:5

(1) Which of the numbers $5, -7, \frac{2}{3}, -\frac{1}{2}$ is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False: $\frac{1}{2} > -8$. **(3)** Explain why $-(-0.2) = 0.2$ makes sense.

Tasks **7:6 Car A and Car B** and **7:8 Oil Business** involve unit rates.

In later grades, **8:1 Xavier's Notes** involves applying the mathematics of rates as a modeling approach in a situation where, as in task 7:12, the actual rate is unlikely to be constant.

In earlier grades, task **6:4 Gas Mileage** involves unit rate calculations. Task **6:3 South Pole Temperatures** features negative temperature values, and task **6:5 Positive and Negative Numbers** concentrates on pre-arithmetic properties of signed rational numbers.

Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:12 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

7:12 Temperature Change

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

7:13 Wire Circle

Teacher Notes

Math Milestones™

Central math concepts

One kind of equation that is used in algebra is a *function equation* that presents the rule for a relationship between two covarying quantities in a situation. An example of a function equation might be $C = 250 + 10n$, where n is the number of cell phone minutes used in a month and C is the resulting monthly charge. (See tasks **6:6 Planting Corn**, **7:6 Car A and Car B** part (3), **8:7 Flight Times and Distances**, and **8:9 Water Evaporation Model** for additional examples).

Another kind of equation is a *constraint equation*. A constraint equation states a condition that must be satisfied. A constraint equation can be viewed as asking a question: Which values from a specified set, if any, make the equation true? Solving a constraint equation is a process of reasoning resulting in a complete answer to that question. An example of a constraint equation might be $250 + 10n = 1000$. Does any positive value of n make this equation true, and if so, what are the value(s) of n that make the equation true?

Function equations and constraint equations differ in an important way. Whereas a constraint equation poses a question about what its solutions are, a function equation doesn't pose a question. Function equations aren't asking, they're telling: telling you the rule for how one quantity depends on another. The function equation $C = 250 + 10n$ expresses a rule for finding C, given any n. By contrast, the constraint equation $250 + 10n = 1000$ is something like a puzzle: what value(s) of n make the equation true? Constraint equations invite you to unravel them, to root out the unknown value(s) of the quantity or quantities they determine yet conceal.

An important point of connection between function equations and constraint equations is that building both kinds of equations requires applying operations to a variable in order to build an expression. One calculates with the variable as if it were a number, applying the meanings and properties of operations. In the case of a function equation, the expression built up in this way defines the rule for the function. The expression $250 + 10n$ defines the rule for the monthly charge given an input number of minutes, n. Meanwhile, in the case of a constraint equation, it often happens that some quantity in the problem can be calculated by two different routes, producing two inequivalent expressions that must nevertheless have the same value. The statement that these two expressions have the same value then becomes a constraint equation for the problem.

For example, a condition might be stated as, "My monthly charge for December was \$100 less than my monthly charge for November because I sent half as many text messages in December compared to November." This rather intricate condition could be represented by the constraint equation $250 + 10(\frac{n}{2}) = 250 + 10n - 100$, where n is the number of text messages sent in November.† Such constraint equations can often be created and solved by thinking functionally: in this example, the stated condition is $C(\frac{n}{2}) = C(n) - 100$.

> 7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

Answer

3.9 in.

Click here for a student-facing version of the task.

Refer to the Standards

7.EE.B.4; MP.4, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

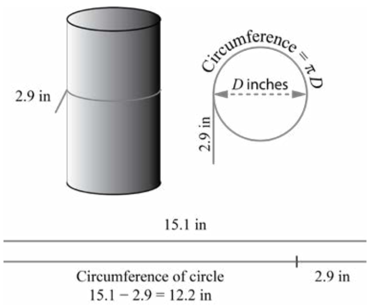
Aspect(s) of rigor:

Application

Additional notes on the design of the task

- Some students may approach the problem by creating and solving a one-variable constraint equation. Some students may use subtraction and division with the given numbers to produce an answer without defining a variable or creating an equation (for a task in which this difference is the explicit topic, see **7:14 Comparing Rose's and Liba's Solutions**). An important discussion would be for students to find correspondences between different approaches, or for students who used one approach to use a classmate's approach, supported by the classmate's explanations.

A wire is bent into a circle, perhaps by bending it around a pole.



In task 7:13, the condition that the total length of the wire must amount to the circumference of the circle plus a leftover amount determines the size of the circle. Defining D to be the circumference of the circle in inches, a constraint equation could be created by naming the circumference of the circle in two ways: (1) as π times the diameter, πD; (2) as 12.2 inches. This leads to the constraint equation $\pi D = 12.2$. Other approaches could include: naming the leftover amount in two ways, as 2.9 inches and as the difference between the total length and the circumference of the circle, leading to the equation $15.1 - \pi D = 2.9$; and expressing the condition that "The length of the wire can be decomposed into the circular part and the leftover part" as the equation $15.1 = \pi D + 2.9$.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: understanding circumference as a length measurement of the perimeter of a circle; defining a variable and building an expression by calculating with it as if it were a number; creating a constraint equation to reflect a condition; basing reasoning on diagrams; solving multi-step one-variable equations; and calculating with decimals.

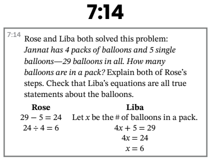
Extending the task

How might students drive the conversation further?

- Students could use estimation to assess whether the answer 3.9 in is reasonable. The diameter of any circle is roughly a third of its circumference, and the circumference is roughly 15 - 3 = 12 in, so the diameter should be roughly a third of 12, or 4 inches. This is close to the answer 3.9 in.
- If students generated different (equivalent) equations, they could show how algebra can be used to transform one equation into another.

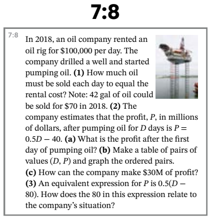
Related Math Milestones tasks

7:14

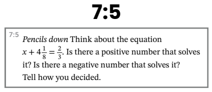

7:14 Rose and Liba both solved this problem: *Jannat has 4 packs of balloons and 5 single balloons—29 balloons in all. How many balloons are in a pack?* Explain both of Rose's steps. Check that Liba's equations are all true statements about the balloons.

Rose	Liba
$29 - 5 = 24$	Let x be the # of balloons in a pack.
$24 \div 4 = 6$	$4x + 5 = 29$
	$4x = 24$
	$x = 6$

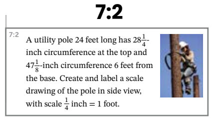
7:8


7:8 In 2018, an oil company rented an oil rig for \$100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for \$70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make \$30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

7:5


7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

7:2


7:2 A utility pole 24 feet long has $28\frac{1}{4}$-inch circumference at the top and $47\frac{1}{8}$-inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale $\frac{1}{4}$ inch = 1 foot.

Task **7:14 Comparing Rose's and Liba's Solutions** and part (2c) of task **7:8 Oil Business** involve solving a constraint equation arising from a stated condition. Task **7:5 Is There a Solution? (Addition)** emphasizes the idea

Additional notes on the design of the task (continued)

- Expressing the given lengths as fractions, an equation model for this task could be written as $15\frac{1}{10} = \pi D + 2\frac{9}{10}$. The equation has exact solution $D = \frac{61}{5\pi}$. (The equation would be easier to solve after multiplying both sides by 10, resulting in $151 = 10\pi D + 29$.)
- Wire is typically thin and deformable, and a length of thin wire could be bent into a circle, for example by wrapping it around a pole as suggested by the diagram. The given numbers in the task, the original length and the leftover length, are accessible to direct measurement while the diameter of the pole would be less accessible to direct measurement.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:13? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:13? In what specific ways do they differ from 7:13?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:13 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

that constraint equations can be interpreted as questions. Task **7:2 Utility Pole Scale Drawing** involves circumference in context.

8:2

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

8:7

8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

8:4

8:4 **(1)** Decide whether each system has exactly one solution, infinitely many solutions, or no solutions. **(2)** For one system, justify your decision to your classmates in two ways: **(a)** drawing graphs of solutions; **(b)** algebraically.

$$\begin{Bmatrix} y = \frac{2}{3}x + 1 \\ y = \frac{2}{3}x + 2 \end{Bmatrix} \quad \begin{Bmatrix} d = 100 - 4t \\ d = 3.5 + t \end{Bmatrix} \quad \begin{Bmatrix} \frac{1}{8}Q + \frac{3}{8}R = -1 \\ Q + 3R = -8 \end{Bmatrix}$$

In later grades, task **8:2 Pottery Factory**, part (2b) of task **8:7 Flight Times and Distances**, and part (4) of task **8:9 Water Evaporation Model** involve solving a constraint equation arising from a stated condition. Task **8:4 System Solutions** involves systems of simultaneous two-variable constraint equations; in one or two of the systems, the forms of the equations invite functional thinking.

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

In earlier grades, **6:1 Charging Cord** and **6:9 Truckload of Gravel** could be solved with an equation of the form $ax = b$, or by dividing appropriately with given numbers. Task **6:13 Is There a Solution? (Multiplication)** emphasizes the idea that constraint equations can be interpreted as questions.

† What number does this condition determine yet conceal?

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

7:14 Comparing Rose's and Liba's Solutions

Teacher Notes

Math Milestones™

Central math concepts

The essential distinction between Rose's and Liba's solutions to the balloon problem is that Rose's solution is *arithmetic* whereas Liba's solution is *algebraic*. Mathematician and mathematics educator Roger Howe, who first used the balloon problem to illustrate this distinction, described it this way:

> "One solution is arithmetic, in the sense that it uses no variables, but simply makes a succession of calculations based on the information in the problem. The other is algebraic. It defines variables, creates expressions and formulates equations in those variables. Then it solves the equations using the standard moves of elementary algebra." (p. 1)
>
> Roger Howe, "From Arithmetic to Algebra"

The arithmetic and algebraic solutions are both mathematically valid. For problems that aren't very complex, the arithmetic approach might be faster and more reliable than the algebraic approach, to the extent that arithmetic is settled knowledge whereas algebra is a new field of study in middle school. For more complex problems, however, the algebraic approach might be the only approach that successfully unravels the problem to find the solution. Algebra can be used to solve problems and create mathematical models for situations where arithmetic alone won't suffice. So it is important for middle-grades students' mathematical futures that they make the transition from arithmetic to algebra. In task 7:14, a comparison of arithmetic and algebraic approaches creates opportunities for reflecting on that transition.

Comparing the two approaches sheds light on both. For example, observe that the operations Liba used to *build* an equation are the reverse of the operations used by Rose. Rose first subtracted, then divided; whereas Liba built an equation by first multiplying, then adding. But then observe that the operations Liba used to solve the equation were the same as the operations by Rose: subtraction, then division. Another thing to notice is that Rose likely calculated 24 for a reason, namely to find out how many balloons are packaged up. By contrast, in Liba's approach the balloon problem can be solved without ever thinking about that particular quantity in the situation: the 24 could just arise as part of a procedural step when carrying out the standard moves of solving an equation. In that sense, the arithmetic solution could be said to probe the quantities in the situation more attentively. Therefore both approaches have mathematical depth and reflect problem-solving power, though the algebraic approach is the one that points the way toward the further mathematics of grade 8 and high school.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: basic meanings of the

7:14 Rose and Liba both solved this problem: *Jannat has 4 packs of balloons and 5 single balloons—29 balloons in all. How many balloons are in a pack?* Explain both of Rose's steps. Check that Liba's equations are all true statements about the balloons.

Rose	**Liba**
$29 - 5 = 24$	Let x be the # of balloons in a pack.
$24 \div 4 = 6$	$4x + 5 = 29$
	$4x = 24$
	$x = 6$

Answer

(1) Explanations of Rose's steps will vary but should include: a reason or purpose why Rose subtracted 29 - 5; what quantity in the situation the result 24 refers to; a reason or purpose why Rose divided 24 ÷ 4; and what quantity in the situation the result 6 refers to. **(2)** Liba's first equation is a true statement about the balloons because if you multiply the number of balloons in a pack (4) by the number of packs (x), then add the 5 single balloons, you get the total number of balloons (29). Liba's second equation is a true statement about the balloons because it follows from the first equation that if you take away the 5 single balloons, the balloons in packs will total 24. Liba's third equation is a true statement about the balloons because it follows from the second equation that the unknown factor must be 6.

Click here for a student-facing version of the task.

Refer to the Standards

7.EE.B.4; MP.2, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

operations of addition, subtraction, multiplication, and division; and mental calculation.

Extending the task

How might students drive the conversation further?

- Students could create an equation of the same form as in task 7:14, such as $3x - 5 = 2$, create a corresponding word problem, and then solve the word problem using both the algebraic and arithmetic approaches.

Related Math Milestones tasks

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

8:1

8:1 Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. Assuming a linear model, about how many more hours will it take him to finish summarizing?

8:2

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

6:1

6:1 $\frac{2}{3}$ of a charging cord is $\frac{1}{2}$ meter long. How long is the charging cord? (Answer in meters.)

6:9

6:9 How much of a $\frac{3}{4}$-ton truckload is $\frac{2}{3}$ ton of gravel?

Task **7:13 Wire Circle** involves a situation with quantitative relationships analogous to the balloon problem, which means that task 7:13 could be used to anchor a discussion like the one comparing Rose's and Liba's solutions.

In later grades, task **8:1 Xavier's Notes** involves a situation in which arithmetic and algebraic approaches are both available. Task **8:2 Pottery Factory** involves an opportunity to build and solve a constraint equation.

In earlier grades, some students might approach task **6:1 Charging Cord** algebraically while others might approach the task arithmetically; the same could be said of task **6:9 Truckload of Gravel**.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- Task 7:14 has a problem-within-a-problem structure that may be unfamiliar. Students could potentially warm up for task 7:14 by first solving the balloon problem.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 7:14? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 7:14? In what specific ways do they differ from 7:14?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 7:14 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

7:14 Comparing Rose's and Liba's Solutions
Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?