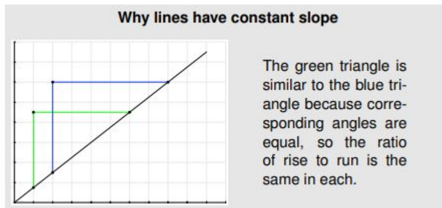
8:10 Missing Coordinate

Teacher Notes

Central math concepts

This task is most directly about the connections between proportional relationships and non-vertical lines in the coordinate plane. With regard to those connections, the most essential fact is that **the slope of such a line has the same value regardless of which two points are chosen to calculate it.** (See the figure.)



The ratio of rise to run is the same for the green and red triangles because the triangles are similar (by the angle-angle criterion for triangle similarity). *Image from*: Common Core Standards Writing Team. (2011). *Progressions for the Common Core State Standards for Mathematics: 6–8, Expressions and Equations* (Draft, 4/22/2011), p. 12. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

The fact that the slope has the same value regardless of which two points are used to calculate it could be used directly as one method for determining the missing coordinate in task 8:10, by writing and solving an equation such as $\frac{(y - 5)}{(5.2 - 5)} = \frac{(7 - 5)}{(6 - 5)}$. Related reasoning might involve completing the figure by drawing two right triangles (which are similar) and making use of the equal side ratios.

Another connection to proportional relationships would involve reasoning that since 5.2 is "20% of the way from 5 to 6," the missing y-coordinate should be "20% of the way from 5 to 7." And since 20% of a doubly long distance must be doubly long, the increment from 5 to 5.2 in the x-coordinate will be matched by an increment from 5 to 5.4 in the y-coordinate. (0.4 is twice as much as 0.2.)

Equations aren't necessary for solving task 8:10, but students could use the equation of a line to find the missing coordinate. Indeed, the fact that the slope has the same value regardless of which two points are used to calculate it underlies all the work students do with the equation of a line. To see this, imagine calculating the slope of a non-vertical line as follows. First, choose a variable point on the line with coordinates (x, y). Next, choose a fixed-but-arbitrary point and denote its coordinates by (x_1, y_1). Then we can calculate two lengths, $y - y_1$ and $x - x_1$. As long as $x \neq x_1$, we can calculate the slope as the ratio of these two lengths:

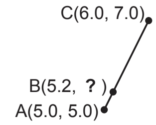
$$\frac{\text{rise}}{\text{run}} = \frac{y - y_1}{x - x_1}$$

Because the value of this ratio will be independent of the chosen points, we have

$$\frac{y - y_1}{x - x_1} = m$$

where m is a number that doesn't depend on y, y_1, x, or x_1, but only depends on the line itself. This is one form of the equation of the line, though perhaps not the form most commonly used. Using algebra, we can rewrite the equation in many different ways, such as $y - y_1 = m(x - x_1)$, $y = mx + (y_1 - mx_1)$, or $y = mx + b$.

8:10 Points A, B, and C lie on a straight line in the coordinate plane. By two methods, find the missing vertical coordinate.



Answer

5.4 (methods may vary).

Click here for a student-facing version of the task.

Refer to the Standards

8.EE.B; MP.1, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- The lack of gridlines in the diagram is intentional in order to encourage working with the numbers directly instead of counting grid units. However, students who are having trouble getting started might be advised to transfer the diagram carefully to a coordinate grid, or to carefully draw some gridlines on the diagram.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 8:10? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 8:10? In what specific ways do they differ from 8:10?

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: determining horizontal and vertical distances between points in the coordinate plane; finding an unknown side length in a triangle, given all side lengths in a triangle known to be similar; finding lengths in a scale drawing; and simple mental calculations involving decimal numbers.

Extending the task

How might students drive the conversation further?

- If students have worked with geometric transformations, they could translate points A, B, and C so that A moves to the origin. This corresponds to subtracting 5 from every coordinate value in the problem. Then the coordinates of the transformed points are (0, 0), (0.2, ?), and (1, 2). The transformed problem may be easier to solve for the coordinates (0.2, 0.4). Now translating back to the original positions by adding 5 to every coordinate value, point B in its original position will have coordinates (5.2, 5.4). Transforming a problem to make it easier to solve is one of the most powerful mathematical practices of all.
- If some students used the two-point form of the equation of a line to determine the equation of the line through points A and C, $y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1) \Rightarrow y = 2x - 5$, while other students used slope reasoning or similar triangles reasoning to develop an equation for the unknown coordinate, such as $\frac{(y - 5)}{(5.2 - 5)} = \frac{2}{1}$, then students could discuss correspondences between the two approaches. For example, instead of solving the equation by the fastest route as $\frac{(y - 5)}{(0.2)} = 2 \Rightarrow y - 5 = 0.4 \Rightarrow y = 5.4$, suppose we solved it this way: $\frac{(y - 5)}{(5.2 - 5)} = \frac{2}{1} \Rightarrow y - 5 = 2(5.2 - 5) \Rightarrow y = 2(5.2) - 10 + 5 \Rightarrow y = 2(5.2) - 5$. This last equation matches up with the equation of the line, $y = 2x - 5$, where $x = 5.2$.

Related Math Milestones tasks

8:7

8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

8:4

8:4 **(1)** Decide whether each system has exactly one solution, infinitely many solutions, or no solutions. **(2)** For one system, justify your decision to your classmates in two ways: **(a)** drawing graphs of solutions; **(b)** algebraically.

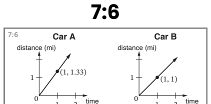
$\begin{cases} y = \frac{2}{3}x + 1 \\ y = \frac{2}{3}x + 2 \end{cases}$ $\begin{cases} d = 100 - 4t \\ d = 3.5 + t \end{cases}$ $\begin{cases} \frac{1}{8}Q + \frac{3}{8}R = -1 \\ Q + 3R = -8 \end{cases}$

7:2

7:2 A utility pole 24 feet long has $28\frac{1}{4}$-inch circumference at the top and $47\frac{1}{8}$-inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale $\frac{1}{4}$ inch = 1 foot.

7:6

7:6



Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

Math Milestones task **8:7 Flight Times and Distances** presents pairs of values that fall along a straight line when graphed. (The constant slope in this context can be interpreted as a speed.) Math Milestones task **8:4 System Solutions** involves two-variable equations.

In earlier grades, Math Milestones task **7:2 Utility Pole Scale Drawing** involves creating a scale drawing, and Math Milestones task **7:6 Car A and Car B** involves proportional relationships that graph as straight lines.

Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:10 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?