8:11 Angle-Angle Similarity Proof

Teacher Notes

Math Milestones™

Central math concepts

Some students may have experiences with touch screen displays in or out of school when using electronic devices for such purposes as map navigation, internet browsing, playing video games, or editing photos. Interacting with touch screens could involve dragging an object, "flipping an image left-right," rotating a map, or pinching-to-zoom on a photo. These operations are reminiscent of the translations, reflections, rotations, and dilations that students study in geometry and that are the basis for careful definitions of congruence and similarity.

In particular, two figures are similar if they can be made to coincide by a sequence of rigid transformations and dilations. By definition, a dilation with center O and positive scale factor r takes points other than O to points that are r times as far from O as they originally were. The figure shows the effect of a dilation with center O and scale factor $r > 1$. The dilation takes point P to point Q, which is located r times as far away from O as point P, in the direction defined by ray OP. By the looks of the figure, the scale factor of this dilation is approximately $r \approx 1.5$.

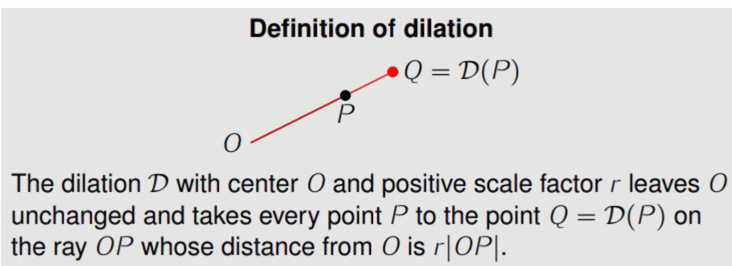


Image from: Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 7–8, HS, Geometry.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona, p. 16.

Similarity links shape to proportionality. A dilation with positive scale factor r will take a segment AB of length $|AB|$ to a dilated segment with length $r|AB|$. If two segments have a certain length ratio, then the dilated segments will have the same length ratio. This also guarantees that dilations preserve angle measure. More generally, shape is preserved under dilations, but size is not preserved unless the scale factor equals 1.

The Angle-Angle criterion for triangle similarity is the statement that if two angles of a triangle are congruent to two angles of another triangle, then the two triangles are similar. To prove the Angle-Angle criterion using transformations, one examines a figure showing two triangles that are arbitrary except for having two congruent angles, and one describes a sequence of rigid transformations and dilations that place one triangle directly on top of the other.

Students can apply the similarity criterion to understand the concept of the slope of a line. The slope of a non-vertical line in the coordinate plane is the same when calculated between any two distinct points, which is a consequence of the line's straightness. The role of geometry in defining

8:11 Study a proof of the Angle-Angle criterion for triangle similarity. Explain one step of the proof in your own words.

Answer

Answers may vary, depending on the proof chosen and the step of the proof chosen. The resource page for task 8:11 has a link to a proof that could be used. Note: the discussion in this Teacher Note is based on geometric transformations (CCSS 8.G.A), but a Euclidean proof could be used for task 8:11.

Click here for a student-facing version of the task.

Refer to the Standards

8.G.A.5; MP.3, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- The task does not require that students construct a proof, only that they examine a proof to understand and explain a step of the argument.
- The resource page can provide a link to a proof of the Angle-Angle similarity criterion. Different proofs could be used, and suggestions are welcome for additional proofs to add to the resource page.

slope is described in the *Progression* document for 7–8 and High School Geometry† and in the *Progression* document for 6–8 Expressions and Equations‡ (see also the Teacher Note for **8:10 Missing Coordinate**).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: applying transformations; using ratio thinking; practicing geometry notation; using definitions; and constructing mathematical arguments.

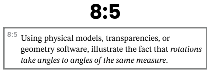
Extending the task

How might students drive the conversation further?

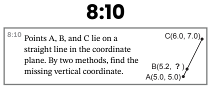
- Students could realize that if triangle *ABC* and another triangle *DEF* have two congruent angles, then in fact they must have three congruent angles, because the measures of the angles of a triangle sum to 180°.
- Because of this, the Angle-Angle criterion for triangle similarity could have been called the Angle-Angle-Angle criterion for triangle similarity. What might be a reason to prefer using a criterion that refers to two angles?
- Students could decide whether two triangles that are congruent will sometimes, always, or never satisfy the Angle-Angle criterion for triangle similarity.

Related Math Milestones tasks

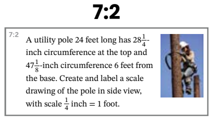
8:5


8:5 Using physical models, transparencies, or geometry software, illustrate the fact that *rotations take angles to angles of the same measure.*

8:10


8:10 Points A, B, and C lie on a straight line in the coordinate plane. By two methods, find the missing vertical coordinate.

7:2


7:2 A utility pole 24 feet long has $28\frac{1}{4}$-inch circumference at the top and $47\frac{1}{8}$-inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale $\frac{1}{4}$ inch = 1 foot.

Task **8:5 Rotations Preserve Angle Measure** focuses on a property of rotations that figures into the logic of an angle-angle similarity argument. Task **8:10 Missing Coordinate** offers the opportunity to connect slope to similarity.

In earlier grades, task **7:2 Utility Pole Scale Drawing** implicitly involves similarity and explicitly involves length ratios.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 8:11? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 8:11? In what specific ways do they differ from 8:11?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2016, March 24). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 7–8, HS, Geometry*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

‡ Common Core Standards Writing Team. (2011). *Progressions for the Common Core State Standards for Mathematics: 6–8, Expressions and Equations* (Draft, 4/22/2011), p. 12. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

