8:12 Fish Tank Design

Teacher Notes

Central math concepts

A common form of mathematical modeling is using mathematics to create a design that satisfies a set of real-world constraints. The constraints in a design problem might arise from finite resources, space requirements, basic physics, even customer preferences. In some cases, the goal of the mathematical model is to find an optimal solution given the constraints. In other cases, any design that satisfies the constraints is acceptable.

Modeling tasks frequently make use of mathematics that was first learned in previous grades, such as unit rates in the case of task 8:12. Mass density is a kind of unit rate; the mass density of water is approximately 1,000 when measured in units of kilograms per cubic meter. Multiplying a mass density in kilograms per cubic meter by a volume in cubic meters results in a mass in kilograms, just as multiplying a speed in meters per second by a time in seconds results in a distance in meters. Other useful examples of density include area densities such as the population density of a city (measured in persons per square mile or persons per square kilometer) or the intensity of sunlight: the solar power flowing into each square meter of a solar cell can be as high as 1,000 watts.

Modeling typically involves a high degree of student agency, because there are more choices for students to make in such tasks, and because the necessary mathematical tools aren't conveniently assembled for the job beforehand. In task 8:12 for example, volume formulas aren't given. Students can use the internet to find formulas they need for geometric measures, and/or they can reason that the problem of finding the volume of a cylinder amounts to finding the area of its base and multiplying that area by the cylinder height. But even with a cylinder formula in hand, the task is designed so that students will still need agency to devise a way to calculate the volume of a quarter-cylinder wedge shape. For that matter, the word "volume" is nowhere stated in the task. Rather, the unit m^3 is a signal about the key quantity; modeling often involves noticing and reasoning about the units of measure in a situation.

For more information and resources about mathematical modeling, see the Modeling section in the digital Coherence Map on achievethecore.org.

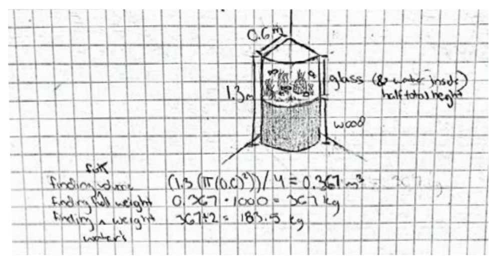
Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: drawing to scale; converting between meters and customary units of length units (feet and/or inches); working with compound units such as m^3; using unit rates; and applying the formula for volume of a cylinder.

8:12 Design a fish tank that fits into the corner of a room. Use a quarter of a cylinder as a model for the tank. To share your design, make a diagram showing the tank measurements. Also, calculate the weight of the water when your tank is filled (1 m^3 of water weighs about 1,000 kg). Write your calculation steps so that a classmate could understand how you did it.

Answer

Answers may vary. The figure shows an example solution by an 8th-grade student. The student began by making a mistake (taking the height of the water to be 1.3 m, twice the actual height) but then corrected the mistake in the final step (dividing by two).



Click here for a student-facing version of the task.

Refer to the Standards

8.G.C.9; MP.4, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Extending the task

How might students drive the conversation further?

- Students could use algebra to show connections between the formulas they used: for example,
  $\frac{1}{4}V_{\text{cylinder}} = \frac{1}{4}(\pi r^2 h) = (\frac{1}{4}\pi r^2)h = (\frac{1}{4}\text{ circle area})\cdot(\text{height})$.
- Students could go online for conversions necessary to calculate how many gallons of water will be in the tank they designed.
- Students could extend the design process to include designing a base for the tank.
- Students could go online to find out more about the hobby and profession of being an aquarist.

Related Math Milestones tasks

8:1

8:1 Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. Assuming a linear model, about how many more hours will it take him to finish summarizing?

8:2

8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

8:7

8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

7:2

7:2 A utility pole 24 feet long has $28\frac{1}{4}$-inch circumference at the top and $47\frac{1}{8}$-inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale $\frac{1}{4}$ inch = 1 foot.

Applied unit rates are implicit or explicit in tasks **8:1 Xavier's Notes**, **8:2 Pottery Factory**, **8:7 Flight Times and Distances**, and **8:9 Water Evaporation Model**.

In earlier grades, task **7:2 Utility Pole Scale Drawing** involves geometric measures and scaling.

Additional notes on the design of the task

- The first modeling challenge in the task is to visualize the shape of a "quarter cylinder." This involves both reading comprehension (inferring the shape from the phrase "fits in the corner") and agency (because imagining the shape is up to the student).
- People can sometimes be surprised by the heaviness of a quantity of water. A cube of water measuring 1 m on a side weighs a thousand kilograms, or over 2,000 pounds. It's very possible to design a tank that would be too heavy for its base, which is why the task requires a calculation of the weight of the water in the tank. (In practice, owners of large fish tanks are sometimes advised to position the tank over two floor joists.)
- The task specifies that students "write your calculation steps so that a classmate could understand how you did it." This is a simple version of the "Report" step of the Modeling cycle. (When a modeling task involves optimizing or decision-making, the Report step could be a more substantial work-product such as a typed document or slide presentation.)

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 8:12? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 8:12? In what specific ways do they differ from 8:12?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:12 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?