8:1 Xavier's Notes

Teacher Notes

Central math concepts

In a situation with two quantities, "assuming a linear model" means assuming that changes in one quantity are directly proportional to changes in the other quantity (with a fixed constant of proportionality). Said another way, assuming a linear model means assuming that the rate of change of one quantity with respect to another quantity remains constant. Graphically, if we were to imagine graphing ordered pairs for the relationship, then assuming a linear model means assuming that the slope would be the same when calculated using any two chosen points.

In this task specifically, one can calculate Xavier's average rate for summarizing 5 pages between 4:45 pm to 6:00 pm; a linear model would then imply that Xavier will summarize the last 7 pages at the same average rate.

Assuming a linear model can be a useful way to analyze a situation, even if this assumption is only an approximation to the reality. In fact, Xavier's rate of summarizing is unlikely to be mathematically constant; probably there are some pages that can be summarized quickly, while others can only be summarized more slowly. Or Xavier may become impatient towards evening and therefore begin summarizing at a faster rate. One doesn't have to *believe* a linear model in order to find out *what would follow from it*.

Modeling always involves a judgment about when simplifying a situation will or won't yield useful insights. In Xavier's case, assuming a linear model might be useful. For example, using a linear model at 6:00 pm might allow Xavier to draw the conclusion that 1 hour and 45 minutes is going to be too long to continue working on the assignment, and therefore he might purposefully pick up the pace.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: distinguishing between clock readings and elapsed time; calculating a unit rate; finding an unknown quantity in a proportional relationship; graphing ordered pairs; and calculating the slope of a line given two points.

Extending the task

How might students drive the conversation further?

- Students could discuss the possible usefulness of assuming a linear model in this situation, and also detail the simplifications that are being made by that assumption.
- Students could relate task 8:1 to scatter plots, because if we had access to more data about Xavier's progress, we could create a

8:1 Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. Assuming a linear model, about how many more hours will it take him to finish summarizing?

Answer

Assuming a linear model, it will take Xavier about 1 hour and 45 minutes longer to finish summarizing. Any answer from 1.5 hours to 2 hours is reasonable. Answers may be expressed in hours, minutes, or hours and minutes.

Click here for a student-facing version of the task.

Refer to the Standards

8.F.B.4; MP.1, MP.2, MP.4, MP.5, MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Additional notes on the design of the task

- Because students may not be familiar with problems that ask them to assume a linear model, it is fine to explain to the students what it means to "assume a linear model," using terms that will best make sense to the students.

scatter plot showing more pairs of values (with one value in each pair being elapsed time, and the other value being the number of pages remaining). If the scatter plot did not suggest a linear association, then we might reevaluate the choice of a linear model.

Related Math Milestones tasks

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

8:7

8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

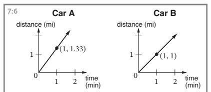
(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

7:12

7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

7:6

7:6



Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Other Math Milestones tasks for Grade 8 that prominently feature linear models or linear functions are **8:9 Water Evaporation Model** and **8:7 Flight Times and Distances**.

In earlier grades, task **7:12 Temperature Change** involves the concept of average rate. Tasks **7:6 Car A and Car B**, **6:6 Planting Corn**, and **6:4 Gas Mileage** prominently feature proportional relationships.

Additional notes on the design of the task (continued)

- The task doesn't require students to use equations, tables, or graphs to represent the linear function model. Students may choose to use any or all of those representations in solving or discussing the problem. Or they may choose to "keep the linear function in their head" and solve the problem by simply performing appropriate numerical calculations such as 12 - 7 = 5, 6 - 4.75 = 1.25, 5/1.25 = 4, 7/4 = 1.75. In that case, students could be asked to describe to a partner what quantities they are calculating (for example, 4 is Xavier's rate in pages per hour), and the partners could look for ways those quantities can be seen in other students' representations.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 8:1? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 8:1? In what specific ways do they differ from 8:1?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

8:1 Xavier's Notes

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?