8:2 Pottery Factory

Teacher Notes

Math Milestones™

Central math concepts

One kind of equation that is used in algebra is a *function equation* that presents the rule for a relationship between two covarying quantities in a situation. An example of a function equation might be $C = 250 + 10n$, where n is the number of cell phone minutes used in a month and C is the resulting monthly charge. (See **6:6 Planting Corn**, **7:6 Car A and Car B** part (3), **8:7 Flight Times and Distances**, and **8:9 Water Evaporation Model** for additional examples).

Another kind of equation is a *constraint equation*. A constraint equation states a condition that must be satisfied. A constraint equation can be viewed as asking a question: Which values from a specified set, if any, make the equation true? Solving a constraint equation is a process of reasoning resulting in a complete answer to that question. An example of a constraint equation might be $250 + 10n = 1000$. Does any positive value of n make this equation true, and if so, what are the value(s) of n that make the equation true?

Function equations and constraint equations differ in an important way. Whereas a constraint equation poses a question about what its solutions are, a function equation doesn't pose a question. Function equations aren't asking, they're telling: telling you the rule for how one quantity depends on another. The function equation $C = 250 + 10n$ expresses a rule for the value of C, given any n. By contrast, the constraint equation $250 + 10n = 1000$ is something like a puzzle: what value(s) of n make the equation true? Constraint equations invite you to unravel them, to root out the unknown value(s) of the quantity or quantities they determine yet conceal.

An important point of connection between function equations and constraint equations is that building both kinds of equations requires applying operations to a variable in order to build an expression. One calculates with the variable as if it were a number, applying the meanings and properties of operations. In the case of a function equation, the expression built up in this way defines the rule for the function. The expression $250 + 10n$ defines the rule for the monthly charge given an input number of minutes, n. Meanwhile, in the case of a constraint equation, it often happens that some quantity in the problem can be calculated by two different routes, producing two inequivalent expressions that must nevertheless have the same value. The statement that these two expressions have the same value then becomes a constraint equation for the problem.

For example, a condition might be stated as, "My monthly charge for December was \$100 less than my monthly charge for November because I sent half as many text messages in December compared to November." This rather intricate condition could be represented by the constraint equation $250 + 10(\frac{n}{2}) = 250 + 10n - 100$, where n is the number of text messages sent in November.[†] Constraint equations can often be created and solved by thinking functionally. In the cell phone example, the stated condition is $C(\frac{n}{2}) = C(n) - 100$.

> 8:2 A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. **(1)** If you start the machines at the same time, which machine will finish its pile first? **(2)** How many min later will the other machine finish its pile? **(3)** Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?

Answer

(1) The fast machine will finish its pile first. **(2)** The slow machine will finish its pile 416 minutes Later. **(3)** 32 pots.

Click here for a student-facing version of the task.

Refer to the Standards

8.EE.C.7b; MP.1, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency, Application

The two expressions surrounding the equal sign in a one-variable constraint equation in a variable x can be interpreted as defining two functions. If the two functions are denoted respectively by f and g, then the constraint equation reads $f(x) = g(x)$. Solving the equation can then be viewed as finding the value(s) of the input variable for which the two functions have equal output values. One way to solve the equation is therefore to graph the equations $y = f(x)$ and $y = g(x)$ on the same set of coordinate axes, and look for points of intersection of the graphs. The x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. This approach is often useful for finding solutions approximately, for example by using technology to graph the functions, make tables of values, or find successive approximations. Using technology can also suggest candidates for exact solutions, which can then be checked algebraically.

In part (3) of task 8:2, a condition must be satisfied: two machines must finish painting at the same time. That condition will determine the value of an unknown quantity, the number of pots that must be moved from the slow machine's pile to the fast machine's pile in order for the condition to be met. A way to think about the constraint equation in functional terms could be to define a function $F = 3p$ for the number of minutes for the fast machine to paint p pots, and define another function $S = 10p$ for the number of minutes for the slow machine to paint p pots; then one way to think about the constraint equation is that it reflects the condition $F(28 + x) = S(50 - x)$, where x is the number of pots moved.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: using unit rates; defining a variable and building an expression by calculating with it as if it were a number; and solving multi-step one-variable equations.

Extending the task

How might students drive the conversation further?

- Mathematics is a powerful tool for finding optimal solutions. Students might wonder, or be asked, about how the original piles of 50 pots and 28 pots should be redistributed between the two machines if the goal is to finish all 78 pots *in the minimum possible time*. (Students may be surprised to discover that the answer has already been found. How might they make sense of this finding?)

Additional notes on the design of the task

- Some students may approach the problem by creating and solving a one-variable constraint equation. Some students may create and solve a system of simultaneous two-variable constraint equations. Some students may use technology such as a spreadsheet to create tables and/or graphs that illuminate how the two machines' finishing times depend on the number of pots moved. An important discussion would be for students to find correspondences between different approaches, or for students who used one approach to use a classmate's approach, supported by the classmate's explanations.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 8:2? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 8:2? In what specific ways do they differ from 8:2?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:2 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Related Math Milestones tasks

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

8:7

8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

8:4

8:4 **(1)** Decide whether each system has exactly one solution, infinitely many solutions, or no solutions. **(2)** For one system, justify your decision to your classmates in two ways: **(a)** drawing graphs of solutions; **(b)** algebraically.

$$\begin{Bmatrix} y = \frac{2}{3}x + 1 \\ y = \frac{2}{3}x + 2 \end{Bmatrix} \quad \begin{Bmatrix} d = 100 - 4t \\ d = 3.5 + t \end{Bmatrix} \quad \begin{Bmatrix} \frac{1}{8}Q + \frac{3}{8}R = -1 \\ Q + 3R = -8 \end{Bmatrix}$$

Part (4) of **8:9 Water Evaporation Model** and part (2b) of **8:7 Flight Times and Distances** both involve solving a constraint equation arising from a stated condition. Task **8:4 System Solutions** involves systems of simultaneous two-variable constraint equations; in one or two of the systems, the forms of the equations invite functional thinking.

7:13

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

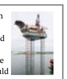
7:14

7:14 Rose and Liba both solved this problem: *Jannat has 4 packs of balloons and 5 single balloons—29 balloons in all. How many balloons are in a pack?* Explain both of Rose's steps. Check that Liba's equations are all true statements about the balloons.

Rose	Liba
$29 - 5 = 24$	Let x be the # of balloons in a pack.
$24 \div 4 = 6$	$4x + 5 = 29$
	$4x = 24$
	$x = 6$

7:8

7:8 In 2018, an oil company rented an oil rig for \$100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for \$70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make \$30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?



6:13

6:13 *Pencils down* Think about the equation $241p = \frac{3}{4}$. Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.

7:5

7:5 *Pencils down* Think about the equation $x + 4\frac{1}{8} = \frac{2}{3}$. Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

In earlier grades, **7:13 Wire Circle**, **7:14 Comparing Rose and Liba's Solutions**, and part (2c) of **7:8 Oil Business** involve solving a constraint equation arising from a stated condition. Tasks **6:13 Is There a Solution? (Multiplication)** and **7:5 Is There a Solution? (Addition)** emphasize the idea that constraint equations can be interpreted as questions.

† What number does this condition determine yet conceal?

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

