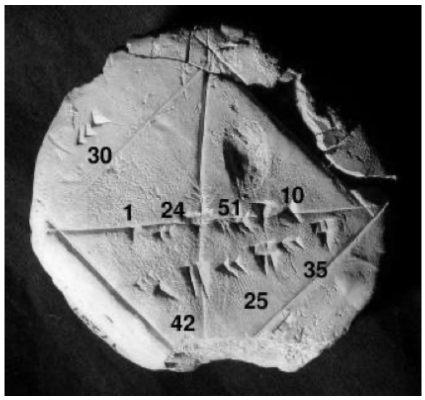
# 8:3 Bicycle Blueprint

Teacher Notes

## Central math concepts

The Pythagorean theorem that extends the concept of number beyond rational numbers has a history long predating the life of Pythagoras himself (c. 570 – c. 495 BC). Special right triangles with three whole-number side lengths were being listed at least as early as c. 1800 BC. Task 8:3 involves a special right triangle with three whole-number side lengths; such triangles are mathematical curiosities, since the square root of a whole number isn't usually a whole number.




By Bill Casselman - Own work, CC BY 2.5, https://commons.wikimedia.org/w/index.php?curid=2154237

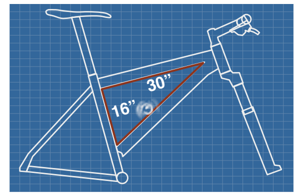
The square root of 2 is a more typical case. The figure shows a clay tablet from around 1800–1600 BC that has been interpreted as giving an approximate value of $\sqrt{2}$ . Note that in a 45-45-90 right triangle with side lengths measuring 1 unit, the hypotenuse will have length $\sqrt{2}$ units. Rounded to thousandths, $\sqrt{2}$ is approximately 1.414, a slight underestimate because $1.414^2 = 1.9994$. Looking at the pattern of the digits in 1.414, it might be natural to guess that the exact value of $\sqrt{2}$ is the repeating decimal $1.\overline{41}$. That would be equivalent to saying that $\sqrt{2} = \frac{140}{99}$, because $\frac{140}{99}$ in decimal form is $1.\overline{41}$. However, the square of $\frac{140}{99}$ is $\frac{19{,}600}{9{,}801}$ which doesn't equal 2.

The clay tablet in the figure gives a better approximation, $\sqrt{2} \approx 1 + \frac{24}{60} + \frac{51}{3{,}600} + \frac{10}{216{,}000}$ or $\sqrt{2} \approx \frac{305{,}470}{216{,}000}$.

Another fraction approximation, used in India in approximately 800–200 BC, was based on the following rule for calculating the length of the hypotenuse of a 45-45-90 right triangle with given side lengths: "Increase the length [of the side] by its third and this third by its own fourth less the thirty-fourth part of that fourth." In other words, multiply the side by $1 + \frac{1}{3} + \frac{1}{3} \times 4 - \frac{1}{3} \times 4 \times 34 = \frac{577}{408}$. However, the square of $\frac{577}{408}$ is $\frac{332{,}929}{166{,}464}$, which doesn't equal 2. And by around 500 BC, it was known that no fraction equals $\sqrt{2}$.

This also implies that there is no length unit we could choose that would give all three length measures of a 45-45-90 triangle as whole numbers. Still, in practical terms physical measurements always have finite precision. In a blueprint, we would never specify a dimension as $\sqrt{2}$, but rather perhaps as a mixed number of units, such as $1\frac{41}{100}$ inches. Thinking of the unit of measure a hundredths of an inch, $1\frac{41}{100}$ becomes a whole number after all (141 hundredths). In that sense, the triangle in task 8:3 isn't

8:3 On this blueprint for building a bike, part of the bike is shaped like a right triangle. The longest side length is illegible because water spilled on the blueprint. Calculate that side length.




### Answer

34"

Click here for a student-facing version of the task.

### Refer to the Standards

8.G.B.7; MP.4, MP.5, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

### Aspect(s) of rigor:

Application

### Additional notes on the design of the task

- The task was inspired by a real-life experience (pre-internet) in which a factory blueprint was defective and an employee was requested who knew the Pythagorean theorem.

### Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 8:3? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 8:3? In what specific ways do they differ from 8:3?

so special after all. Architects, manufacturers, and home crafters alike are working in the end with rational units.

And yet: if we can imagine a square, then we can imagine the diagonal of a square. And if the length of that diagonal is a number, then there have to exist numbers that aren't fractions. A number that isn't a fraction is called irrational—not because the number is unreasonable, but because its value isn't the value of any whole-number ratio.

## Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: understanding the square root symbol; and creating an equation involving the square of an unknown number.

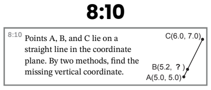
## Extending the task

How might students drive the conversation further?

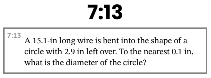
- Students could use the Pythagorean theorem to estimate the percent savings achieved by "cutting a corner" when walking—supposing that the shortcut is the hypotenuse of a right triangle with one side measuring 1 unit and the other side measuring anywhere from 1 to 5 units.

## Related Math Milestones tasks

**8:10**


8:10 Points A, B, and C lie on a straight line in the coordinate plane. By two methods, find the missing vertical coordinate.


**7:13**


7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

The Pythagorean theorem allows distances between points to be calculated when the coordinates of the points are known; as applied to task **8:10 Missing Coordinate**, the theorem could be used to show that segment *AC* is five times as long as segment *AB*. This agrees with the fact that a dilation with center *A* and scale factor 5 takes point *B* to point *C*.

In earlier grades, task **7:13 Wire Circle** involves a decimal approximation to the irrational number $\pi$. (As a historical note to compare with the case of $\sqrt{2}$, the number $\pi$ was [proved to be irrational in 1761](#).)

### Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:3 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

## Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

### Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

### Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

### Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?