8:5 Rotations Preserve Angle Measure

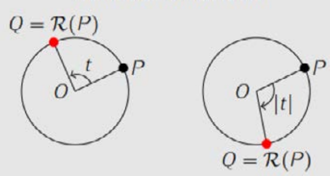
Teacher Notes

Central math concepts

Some students may have experiences with touch screen displays in or out of school when using electronic devices for such purposes as map navigation, internet browsing, playing video games, or editing photos. Interacting with touch screens could involve dragging an object, "flipping an image left-right," rotating a map, or pinching-to-zoom on a photo. These operations are reminiscent of the translations, reflections, rotations, and dilations that students study in geometry and that are the basis for careful definitions of congruence and similarity.

Rotations in particular are defined as shown in the figure, which shows the effect of a rotation around the point *O* through an angle of measure *t*° in two cases, $t \geq 0$ and $t < 0$. In particular, in the case $t > 0$, the rotation takes point *P* clockwise to point *Q*, which is located the same distance from *O* as point *P* and forms angle ∠*POQ* with measure *t*°.†

Definition of rotation



The rotation $\mathcal{R}$ around the point O through the angle t takes a point P to the point $Q = \mathcal{R}(P)$ as follows. If $P = O$, then $\mathcal{R}(O) = O$. If $P \neq O$ and $t \geqslant 0$, then Q is on the circle with center O and radius $|OP|$ so that $\angle POQ = t^\circ$ and Q is counterclockwise from P. If $t < 0$, we rotate clockwise by $|t|^\circ$.

Image from: Common Core Standards Writing Team. (2016, March 24). Progressions for the Common Core State Standards in Mathematics (draft). Grades 7–8, HS, Geometry. Tucson, AZ: Institute for Mathematics and Education, University of Arizona, p. 14.

Rotations, along with reflections and translations, are rigid motions. The following properties of rigid motions are assumed as axioms:

1. Rigid motions map lines to lines, rays to rays, and segments to segments.
2. Rigid motions preserve distance.
3. Rigid motions preserve angle measure.

The definition of a rotation emphasizes that a rotation through a nonzero angle moves all points of the plane other than the center of rotation. Therefore a rotation doesn't just move a triangle, or a line segment, or other figure; rather, the rotation moves the points of the plane, and because geometric figures are made of those moving points, the figures move too. As the Geometry Progression document notes,

> When the transformation is a rigid motion (a translation, rotation, or reflection) it is useful to represent it using transparencies because two copies of the plane are represented, one by the piece of paper and one by the transparency. These correspond to the domain and range of the transformation, and emphasize that

> 8:5 Using physical models, transparencies, or geometry software, illustrate the fact that *rotations take angles to angles of the same measure*.

Answer

Answers may vary but illustrations should identify both that rotations take angles to angles, and that rotations take angles to angles of the same measure.

Click here for a student-facing version of the task.

Refer to the Standards

8.G.A.1; MP.3, MP.5, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- Because it is an axiom that rotations preserve angle, the intent of task 8:5 isn't to have students prove that rotations preserve angle. Rather, by illustrating the fact, students build a base of experience for reasoning with the axioms.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 8:5? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to 8:5? In what specific ways do they differ from 8:5?

> the transformation acts on the entire plane, taking each point to another point. The fact that rigid motions preserve distance and angle is clearly represented because the transparency is not torn or distorted. (p. 14)

In high school, students will learn more explicitly that transformations are functions from the plane to itself.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: applying transformations; practicing geometry notation; using definitions; and constructing mathematical arguments.

Extending the task

How might students drive the conversation further?

- Students could draw consequences from the properties of rotations. For example, if a square with area 100 square units is rotated, what is the area of the rotated square? Justify your answer carefully using properties of rotations.

Related Math Milestones tasks

8:11

8:11 Study a proof of the Angle-Angle criterion for triangle similarity. Explain one step of the proof in your own words.

7:10

7:10 In ΔABC, side AB is 4 units long, side BC is 3 units long, and angle A measures 30°. Sketch two ways ΔABC might look.

Task **8:11 Angle-Angle Similarity Proof** concerns a proof that (in the most general case) relies upon the angle-measure preserving property of rotations.

In earlier grades, task **7:10 Triangle Conditions** involves conditions under which one, more than one, or no triangle may be constructed; these considerations underlie the triangle congruence theorems proved using transformations.

Curriculum connection (continued)

2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:5 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† The sense of a rotation is sometimes characterized by a "right-hand rule": for a rotation through a positive angle, if you place the edge of your right hand along segment *OP* with your thumb pointing up from the paper, your fingers will sweep from *P* to *Q*.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?