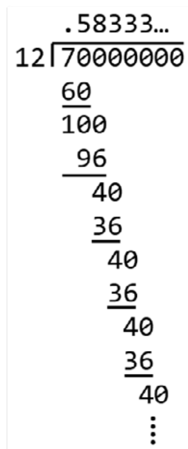
8:6 Rational Form

Teacher Notes

Math Milestones™

Central math concepts

Part (1). Every fraction $\frac{a}{b}$ can be expressed as a decimal that terminates or eventually repeats, and conversely every terminating or repeating decimal can be expressed as a fraction. To find the decimal expansion of a fraction $\frac{a}{b}$, one could perform the long division $a \div b$. For example, $\frac{7}{12} = 7 \div 12$, and if we perform the long division as shown, we eventually find that the process repeats. The result of the process could be expressed as



$\frac{7}{12} = 0.58\overline{3}$.

Looking at the structure of the decimal $0.58\overline{3}$, it appears to be the sum of 0.58 and $0.00\overline{3}$, which is to say the sum of $\frac{58}{100}$ and $\frac{1}{100}$ of $\frac{1}{3}$. Let's see if that fraction sum indeed equals $\frac{7}{12}$:

$$\frac{58}{100} + \frac{1}{100} \times \frac{1}{3}$$
$$= \frac{58}{100} + \frac{1}{300}$$
$$= \frac{174}{300} + \frac{1}{300}$$
$$= \frac{175}{300}$$
$$= \frac{7}{12}.$$

Thus we have traveled full circle from the fraction to the decimal back to the fraction.

Similarly, in part (a) of task 8:6, the number $1.041\overline{6}$ could be seen as

$$1 + \frac{4}{100} + 0.001\overline{6}$$
$$1 + \frac{4}{100} + \frac{1}{6} \div 100$$
$$1 + \frac{4}{100} + \frac{1}{600}$$
$$1 + \frac{24}{600} + \frac{1}{600}$$
$$1 + \frac{25}{600}$$
$$1 + \frac{1}{24}$$
$$\frac{25}{24}.$$

8:6 Write as a fraction in lowest terms: **(1)** $1.041\overline{6}$. **(2)** $3^2 \cdot 3^{-5}$.

Answer

(1) $\frac{25}{24}$. **(2)** $\frac{1}{27}$.

Click here for a student-facing version of the task.

Refer to the Standards

8.NS.A.1, 8.EE.A.1; MP.6, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- There are many ways to approach part (a) of the task. Students could experiment with a calculator to decide that $1000 \times 0.000\overline{6} = \frac{2}{3}$ or recognize that $0.000\overline{6} = \frac{2}{3} \div 1000$.
- Whatever the approach, carrying out the long division will settle the question of whether $\frac{25}{24}$ equals $1.041\overline{6}$.
- The intent of part (a) is to involve students in thinking synthetically about fractions, decimals, place value, and their relationships; it is not a procedural fluency task. (Part (b) could be considered a procedural fluency task after the properties of integer exponents are understood.)

Part (2). Students extend the meaning of exponents from positive exponents to integer exponents based on the fundamental property $x^a x^b = x^{a+b}$. This property summarizes the patterns in positive exponents that students are familiar with from previous grades, for example $7^3 7^2 = (7{\cdot}7{\cdot}7)(7{\cdot}7) = 7^5$ and $7^2 = 7^{1+1} = 7^1{\cdot}7^1 = 7{\cdot}7$. The property also gives meaning to negative exponents and the exponent 0. For example, consider the question of what we take 7^0 to be. Suppose initially we believe we should take 7^0 to be 0. The fundamental property has something to say about that choice, because it would follow that

Using the property here

$$7^2 = 7^{2+0} = 7^2\, 7^0 = 7^2 \cdot 0 = 0.$$

Thus if we take 7^0 to be zero, we will also have to take 7^2 to be zero. But the familiar meaning of 7^2 is $7{\cdot}7 = 49$, so taking 7^0 to be 0 wouldn't be using the fundamental property to extend familiar meanings of exponents.

If we aren't going to take 7^0 to be zero, then could we take it to be something else? Perhaps 7^0 could be any positive number we like? To find out what's possible, let's apply the fundamental property again:

$7^0\, 7^0 = 7^{0+0}$ (using the property with both exponents zero)

$7^0\, 7^0 = 7^0$. (because 0 + 0 = 0)

Dividing both sides of the last equation by 7^0 (remember that 7^0 is nonzero), we find

$7^0 = 1$.

So when using the fundamental principle to extend the meaning of exponents from positive to integer exponents, there is only one possibility for the value of 7^0, which is 1. And the "7" here could have been any nonzero number, so expressing the conclusion generally, x^0 equals 1 for any nonzero number x. This consequence of the fundamental property is sometimes called "the zero power rule."

Another consequence of the fundamental property is sometimes called "the negative exponent rule," $x^{-a} = \frac{1}{x^a}$. This rule follows from the fundamental property by applying the fundamental property with equal and opposite exponents: $x^a x^{-a} = x^{a+-a} = x^0$. Remembering that $x^0 = 1$, we have $x^a x^{-a} = 1$. This says that x^a and x^{-a} are reciprocals, or equivalently that $x^{-a} = \frac{1}{x^a}$. That's "the negative exponent rule." Students working from long lists of integer exponent rules may not realize that those rules are special-purpose applications of a fundamental property, rather than independent principles.

For more observations on integer exponents, the relationship between fractions and repeating decimals, and the beginnings of students' studies of irrational numbers, see the relevant *Progression* document.†

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 8:6? Locate 2–3 similar tasks in that unit. How are the tasks similar to each other, and to 8:6? In what specific ways do they differ from 8:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: adding fractions; remembering fraction-decimal equivalents; understanding place value; practicing long division in simple cases; and calculating mentally.

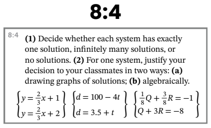
Extending the task

How might students drive the conversation further?

- Students could check the answer $\frac{25}{24}$ by calculating 25 ÷ 24 by long division.
- If $d = 0.\overline{6}$ then students could evaluate the claim that $10d = 6 + d$. Taking the equation as true, what do you get if you solve it for d?
- Students could repeat the above with $d = 0.\overline{9}$.

Related Math Milestones tasks

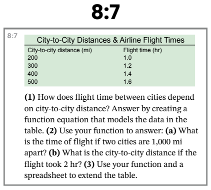
8:4



8:4 **(1)** Decide whether each system has exactly one solution, infinitely many solutions, or no solutions. **(2)** For one system, justify your decision to your classmates in two ways: **(a)** drawing graphs of solutions; **(b)** algebraically.

$\begin{cases} y = \frac{2}{3}x + 1 \\ y = \frac{2}{3}x + 2 \end{cases}$ $\begin{cases} d = 100 - 4t \\ d = 3.5 + t \end{cases}$ $\begin{cases} \frac{1}{8}Q + \frac{3}{8}R = -1 \\ Q + 3R = -8 \end{cases}$

8:7



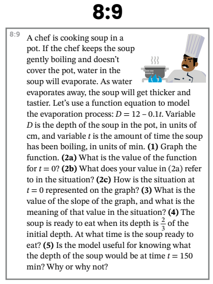
8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

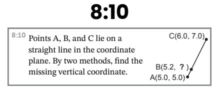
(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

8:9



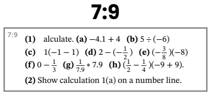
8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

8:10



8:10 Points A, B, and C lie on a straight line in the coordinate plane. By two methods, find the missing vertical coordinate.

7:9



7:9 **(1)** alculate. **(a)** $-4.1 + 4$ **(b)** $5 \div (-6)$ **(c)** $1(-1 - 1)$ **(d)** $2 - (-\frac{1}{2})$ **(e)** $(-\frac{3}{8})(-8)$ **(f)** $0 - \frac{1}{3}$ **(g)** $\frac{1}{7.9} * 7.9$ **(h)** $(\frac{1}{2} - \frac{1}{4})(-9 + 9)$.

(2) Show calculation 1(a) on a number line.

Fractions and decimals appear in several grade 8 tasks including **8:4 System Solutions**, **8:7 Flight TImes and Distances**, **8:9 Water Evaporation Model**, and **8:10 Missing Coordinate**.

In earlier grades, task **7:9 Calculating with Rational Numbers** extends calculation from positive fractions to signed rational numbers.

† Common Core Standards Writing Team. (2013, July 4). *Progressions for the Common Core State Standards in Mathematics (draft). Grades 6–8, The Number System; High School, Number*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Math Milestones™

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?