8:7 Flight Times and Distances

Teacher Notes

Math Milestones™

Central math concepts

In this task the central concept is the linear function. Linear functions are a generalization of proportional relationships. To see this more clearly, let's first look closely at proportional relationships.

In a proportional relationship, two quantities x and y covary in such a way that the ratio y/x is constant. Proportional relationships are functions that have the equation form $y = rx$. If y is proportional to x, then something more is also true, namely, changes in y are proportional to changes in x. This follows from the distributive property, because if y_1 and y_2 are two different y values, then $y_1 = rx_1$ and $y_2 = rx_2$, so the change in quantity y equals

$$y_2 - y_1$$
$$= rx_2 - rx_1$$
$$= r(x_2 - x_1)$$

In other words, the change in y is always a multiple r of the change in x. Another way to say that changes in y are proportional to changes in x is to say that *the rate of change of* y *with respect to* x *is a constant value*. (The constant value is r.)

There are more functions besides proportional relationships in which the rate of change of y with respect to x is constant. In any function that has a function equation of the form $y = mx + b$, changes in y are directly proportional to changes in x, even though y is not proportional to x when b is nonzero. This again follows from the distributive property, because if y_1 and y_2 are two different y values, then $y_1 = mx_1 + b$ and $y_2 = mx_2 + b$, so the change in quantity y equals

$$y_2 - y_1$$
$$= (mx_2 + b) - (mx_1 + b)$$
$$= mx_2 - mx_1$$
$$= m(x_2 - x_1)$$

In other words, the change in y is always a constant multiple m of the change in x. Another way to say that changes in y are proportional to changes in x is to say that *the rate of change of* y *with respect to* x *is a constant value.* (The constant value is m.)

In a situation where one quantity changes at a constant rate with respect to another quantity, we can express the first quantity as a linear function of the second, even if the first quantity isn't directly proportional to the second quantity. In task 8:7, the flight times shown in the table change at a constant rate with respect to city-to-city distances; this can be seen from the fact that the values in the second column increase by 0.2 hours for every increase of 100 miles in the values in the first column. This is why there is a linear function that models the data in the table.

For more discussion of these concepts, see pp. 5, 6 of *Progressions for the Common Core State Standards in Mathematics (draft). Grade 8, High School, Functions* (Common Core Standards Writing Team, March 1, 2013. Tucson, AZ: Institute for Mathematics and Education, University of Arizona) and pp. 11, 12 of *Progressions for the Common Core State Standards in*

8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

Answer

(1) $H = 0.002D + 0.6$, where H is the number of hours of flight time and D is the city-to-city distance in miles. Any function equation that is equivalent to $y = 0.002x + 0.6$ is correct, as long as the equation is consistent with the contextual meanings of both variables and the meanings of both variables are stated explicitly. Examples: $y = (0.2/100)(x - 200) + 1$ is correct, if y is stated to be the number of hours of flight time and x is stated to be the city-to-city distance in miles; $y = 12x + 36$ is correct, if y is the number of minutes of flight time and x is the city-to-city distance in hundreds of miles. **(2) (a)** $H = 0.002(1000) + 0.6 = 2.6$ hr. **(b)** $2 = 0.002D + 0.6 \Rightarrow D = 700$ mi. (Variable names, equations, and units may differ depending on the form of the function equation in part (1). Note, the task requires creating and solving equations - not just calculations with numbers.) **(3)** Answers vary; click here for an example.

Click here for a student-facing version of the task.

Refer to the Standards

8.F; MP.1, MP.2, MP.4, MP.7, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with

[Mathematics (draft). Grades 6–8, Expressions and Equations](#) (*Ibid.*, 22 April 2011).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: calculating unit rates; graphing ordered pairs from a table of values; converting units; creating equations to describe relationships between quantities that covary; solving one-variable linear equations to find an unknown value; and using technology.

Extending the task

How might students drive the conversation further?

- Students could ask or be asked about the meaning of the constants 0.002 and 0.6 in the function equation $H = 0.002D + 0.6$. The coefficient 0.002 is the number of hours taken to fly 1 mile; this value is much less than 1, because planes fly so quickly (it takes much less than 1 hour to fly 1 mile). Meanwhile, the constant 0.6 indicates a flight time of 0.6 hours for a trip of 0 miles; one interpretation of this value would be as a quantity of time that is wasted on the two runways. Be aware that students may be unfamiliar with air travel in their own life experiences or in the movies they have seen, stories they have read, and so on.
- In a situation where one quantity is a linear function of another, it is also true that the second quantity is a linear function of the first. This is because $\Delta y = m\Delta x$ implies $\Delta x = (\frac{1}{m})\Delta y$. Students could ask or be asked about how the city-to-city distance depends on flight time. That linear function can be written $y = 500x - 300$, where y is the city-to-city distance in miles and x is the flight time in hours. Students could discuss the meaning of the coefficient 500 in the context of the situation (it has units of miles per hour and could be interpreted as flight speed).
- Students could calculate the average speed for flights of different distances, observing that the average speed increases for longer trips, because the "startup cost" of 0.6 hours becomes a smaller and smaller fraction of the total trip time.

Related Math Milestones tasks

8:1

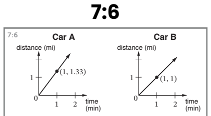
8:1 Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. **Assuming a linear model, about how many more hours will it take him to finish summarizing?**

8:9

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

7:6

7:6



Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

Tasks **8:1 Xavier's Notes** and **8:9 Water Evaporation Model** prominently feature linear models.

In earlier grades, tasks **7:6 Car A and Car B**, **6:6 Planting Corn**, and **6:4 Gas Mileage** prominently feature proportional relationships.

Refer to the Standards (continued)

locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- In part (1), students create a function equation. In part (2), students can use the function equation to answer two questions about particular flight times and distances. If one wished, one could use the function equation to answer many more questions about particular flight times and distances; for that reason, one might say that a function equation contains in a single statement all the infinitely many specific possibilities inherent in a situation with covarying quantities. Part (3) shows the power of a function equation in combination with technology to quickly generate dozens, hundreds, even thousands of those particular possibilities. This power is one reason why it is worth learning how to develop function equations to model situations, instead of only ever solving problems about one unknown value in a particular occurrence of a situation.

Curriculum connection

- In which unit of your curriculum would you expect to find tasks like 8:7? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 8:7? In what specific ways do they differ from 8:7?
- Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:7 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?