8:8 Heart Rate and Exercise

Teacher Notes

Central math concepts

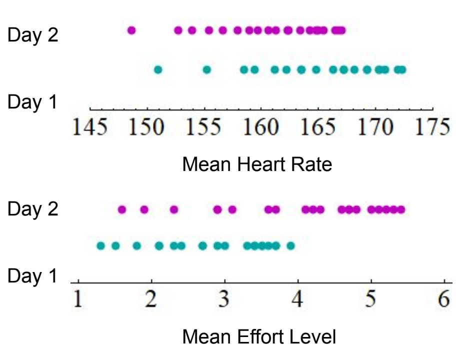
When interpreting a scatter plot, the first order of business is to understand what fact is being represented by a single (x, y) data point. What is the story behind one data point? What process of measurement or experimentation gave rise to the number x and the number y together? It is premature to evaluate trends or patterns in the distribution as a whole without knowing the meaning of a single data point. In task 8:8 for example, the data points were produced during an experiment in exercise physiology:

> A researcher asked people doing exercise to rate their effort level. The researcher also measured people's heart rates. Data were taken on two different days. Each person's heart rate (beats per min.) and effort (1–6 scale) were recorded every 3 min. A group average was then calculated, creating one data point such as (150.9, 1.3).

So the data point (150.9, 1.3) refers to the fact that at a certain moment of time during the training session, the average heart rate among the people doing exercise was 150.9 beats per minute, while the average effort level rating on a 1–6 scale was 1.3 at that same point in time.

Note that because there are 20 data points, and because a new data point was produced every three minutes, one can infer that the data set for each day was the outcome of a 60-minute training session. Apparently, as the training session wore on, people's heart rates increased and so did their subjective level of effort. Qualitatively that was true on both days, but quantitatively that pattern played out differently on Day 1 vs. Day 2.

The circumstance that links the values 150.9 and 1.3 into an ordered pair is that the two measurements were taken at the same time. The values 150.9 and 1.3 are paired into a single observation (150.9, 1.3). However, it is possible and sometimes valuable to "intentionally forget" that the x values and the y values in a data set are paired, by examining the distribution of x values and the distribution of y values separately (see the figure).



In the upper figure, the heart rate data for each day has been plotted on a stacked dot plot. The upper figure shows that mean heart rates were slightly lower on Day 2 (see CCSS 7.SP.B.3). In the lower figure, the effort

8:8 A researcher asked people doing exercise to rate their effort level. The researcher also measured people's heart rates. Data were taken on two different days. **(1)** Use technology to plot the data from both days. (View heart rates in a window from 145 to 175.) Describe the main patterns you see. **(2)** On one of the days, the exercise room was warm, and on the other day, the room was cool. Which day do you think was the warm day? Tell how you decided, and support your answer with calculations.

Heart Rate & Effort in Exercise

Day 1 HR, Effort	Day 2 HR, Effort
150.9, 1.3	148.6, 1.6
155.2, 1.5	152.7, 1.9
158.5, 1.8	153.9, 2.3
159.4, 2.1	155.4, 2.9
161.2, 2.1	156.6, 2.9
162.2, 2.3	157.9, 3.1
163.5, 2.4	158.9, 3.6
163.5, 2.7	159.7, 3.7
164.8, 2.7	160.6, 4.1
166.3, 2.9	161.3, 4.2
167.2, 3.0	162.3, 4.3
167.2, 3.3	162.4, 4.6
168.1, 3.4	163.4, 4.7
169.2, 3.4	164.2, 4.8
169.2, 3.5	164.8, 4.7
170.3, 3.5	165.0, 5.0
170.8, 3.6	165.4, 5.1
170.4, 3.7	167.0, 5.2
171.9, 3.7	166.5, 5.3
172.3, 3.9	166.7, 5.4

Click here to get the data online. Each person's heart rate (beats per min.) and effort (1–6 scale) were recorded every 3 min. A group average was then calculated, creating one data point such as (150.9, 1.3).

Answer

(1) See figure. (A spreadsheet of the data is online here.) **(2)** The data suggest that Day 2 was the warm day and Day 1 was the cool day. Explanations may vary but could include the following observation(s): (i) When it's warmer, you can feel like you're working harder even if your body is doing the same thing. And even though heart rates were a bit lower on Day 2 (average heart rate about 161 bpm vs. 165 bpm on Day 1), people still felt they were working harder on Day 2 (average effort level about 4, vs. 2.8 on Day 1). (ii) A linear model for Day 2 has a steeper slope than a linear model for Day 1. That means on Day 2, raising your heart rate required a greater increase in effort. This again suggests that Day 2 was the warm day.

level data for each day has been plotted on a stacked dot plot. The lower figure shows that mean effort levels were generally higher on Day 2. This is also visible on the scatter plot—not as a steeper slope for Day 2, but rather in the fact that the Day 2 data points are generally shifted vertically upward compared to the Day 1 data points.

The slope of a graph has several important interpretations that recur over and over in applications. One interpretation of slope is geometric: slope measures the steepness of a line in the coordinate plane. Another interpretation of slope is that the value of the slope equals the rate of change for a quantity that varies with time. A third interpretation of slope is as a marginal return: a slope of m means that each unit increase in quantity x corresponds to (or costs, or yields) an increase of amount m in quantity y. (The "increase" is a decrease when m is negative.) In the context of task 8:8, each unit increase in heart rate "costs" an increase in effort. The marginal cost of that increase was greater on the warmer day, statistically speaking.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: thinking about data in context; calculating measures of center and measures of variation; describing patterns in distributions of univariate and bivariate measurement data; and using technology.

Extending the task

How might students drive the conversation further?

- Students could use technology to fit a best-fit line to both data sets and interpret the two slopes in context.
- Students could create a similar experiment of their own to measure some aspect of exercise and the body's responses to exertion, and ask ons about the da

Related Math Milestones tasks

8:1

8:1 Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. Assuming a linear model, about how many more hours will it take him to finish summarizing?

6:3

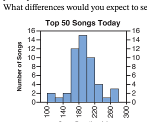
6:3 The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

Time	Hours after Midnight	Temp °F
8:00 pm	–4	–42
9:00 pm	–3	–42
10:00 pm	–2	–41
11:00 pm	–1	–40
Midnight	0	–39
1:00 am	1	–39
2:00 am	2	–38

Plot the data on graph paper and label the plot. Describe any patterns you see.

6:7

6:7 **(1)** Look up the 50 top songs on a music streaming service. Type each song's duration into a spreadsheet. **(2)** Write a sentence about the data giving a measure of center and a measure of variability. **(3)** Make a histogram of the data.* **(4)** Write a sentence describing the overall pattern of the distribution and any striking deviations from the overall pattern. **(5)** Imagine that one year from now, you go back online and repeat (1)–(4). In what ways would you expect the data distribution to look similar? What differences would you expect to see?



*Use this histogram for (4) and (5) if you don't do (3).

Task **8:1 Xavier's Notes** involves given information that could be viewed as a two-point data set of bivariate data.

In earlier grades, task **6:3 South Pole Temperatures** uses the coordinate plane to represent a set of bivariate measurement data, while task **6:7 Song Length Distribution** involves a distribution of univariate data.

Answer (continued)

Click here for a student-facing version of the task.

Refer to the Standards

8.SP.A.1–3; MP.4, MP.5. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Application

Additional notes on the design of the task

- The data for task 8:8 comes from a 2004 undergraduate research study in the field of exercise physiology.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 8:8? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to 8:8? In what specific ways do they differ from 8:8?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:8 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

