8:9 Water Evaporation Model

Teacher Notes

Central math concepts

Initial value and rate of change. Properties of a function can be read from the function equation that defines it. For a function equation of the form $F = c + rt$, two important properties of function F can be read from the equation: its initial value and its rate of change. The initial value is c, as can be seen by evaluating the function at $t = 0$ ($F = c + r{\cdot}0 = c$). To see that the rate of change of the function is r, let's evaluate the function at two arbitrary different times, t_1 and t_2, which leads to corresponding output values $c + rt_1$ and $c + rt_2$. The rate of change of the function is the difference of those two function values divided by the difference in the two times:

$$\text{Rate} = \frac{\text{change in } F}{\text{change in } t} = \frac{(c + rt_2) - (c + rt_1)}{t_2 - t_1}.$$

This simplifies:

$$\frac{c + rt_2 - c - rt_1}{t_2 - t_1} = \frac{rt_2 - rt_1}{t_2 - t_1} = \frac{r(t_2 - t_1)}{t_2 - t_1} = r.$$

Thus, for a function defined by $F = c + rt$, the function's rate of change is r, independently of the interval of time over which the rate of change is calculated. A function defined by a function equation of the form $F = c + rt$ could therefore be called a constant-rate function.

Graph. The graph of a function defined by $F = c + rt$ is the set of all ordered pairs $(t, c + rt)$. These ordered pairs "pair" outputs with inputs. When plotted on a coordinate plane, the ordered pairs create a picture of how quantity F depends on quantity t. Something interesting happens when you calculate the slope of the graph of a constant-rate function $F = c + rt$. Using two arbitrary distinct points on the graph, $(t_1, c + rt_1)$ and $(t_2, c + rt_2)$, the slope can be calculated as

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{(c + rt_2) - (c + rt_1)}{t_2 - t_1} = r$$

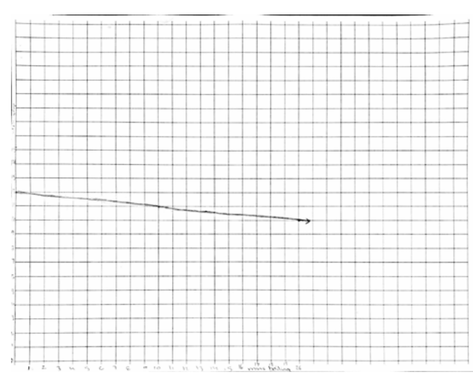
(thanks to the same algebra as before). This shows that the slope of the graph has value r, independently of the two points chosen to calculate it. Not only is the slope of the graph equal to the rate of change of the function, but also the constancy of the slope of the graph implies that the graph of a constant-rate function is a straight line. This justifies our practice of calling constant-rate functions linear.

"The slope of the graph equals the rate of change" is a principle that generalizes far beyond the middle grades. Differential calculus extends this idea to nonlinear functions, for which the slope and rate of change both vary with time.

8:9 A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: $D = 12 - 0.1t$. Variable D is the depth of the soup in the pot, in units of cm, and variable t is the amount of time the soup has been boiling, in units of min. **(1)** Graph the function. **(2a)** What is the value of the function for $t = 0$? **(2b)** What does your value in (2a) refer to in the situation? **(2c)** How is the situation at $t = 0$ represented on the graph? **(3)** What is the value of the slope of the graph, and what is the meaning of that value in the situation? **(4)** The soup is ready to eat when its depth is $\frac{2}{3}$ of the initial depth. At what time is the soup ready to eat? **(5)** Is the model useful for knowing what the depth of the soup would be at time $t = 150$ min? Why or why not?

Answer

(1) See figure. **(2)** **(a)** 12. **(b)** The value 12 refers to the depth of the soup in the pot in centimeters just when it begins to boil. **(c)** The graph includes the point (0, 12), and this point represents the fact that the soup in the pot is 12 cm deep just when it begins to boil. **(3)** The slope of the graph is -0.1, which means that every minute, the depth of the soup in the pot decreases by 0.1 cm. **(4)** The soup is ready to eat after it has been boiling for 40 min. **(5)** The model isn't useful for knowing what the depth of the soup would be at time t = 150 min, because the model says that the depth of the soup is -3 cm at that time, but the depth of the soup cannot be negative.



Click here for a student-facing version of the task.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: graphing a function defined by a function equation; interpreting and using negative numbers in context; working with simple fractions and decimals in context; creating and solving a constraint equation to find an unknown value; and using technology.

Extending the task

How might students drive the conversation further?

- The function equation $D = 12 - 0.1t$ can be solved for $t = 100(12 - D)$. Students could be asked to interpret this function equation: what is the input variable, what is the output variable, and can we make sense of the rule defined by the expression on the right-hand side?
- Supposing that the chef forgets about the soup and leaves it on the stove for 150 minutes, students could sketch a graph showing the depth of the soup from $t = 0$ to $t = 150$ min. This graph would then itself be a mathematical model that applies to the situation over a longer period of time than the function model stated in task 8:9. Students could debate whether the least possible value of the depth variable is reasonably 0 cm (which would imply an empty pot) or reasonably something greater than 0 cm (corresponding to a layer of burnt soup-sludge).

Related Math Milestones tasks

8:1

8:1 Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. Assuming a linear model, about how many more hours will it take him to finish summarizing?

8:7

8:7

City-to-City Distances & Airline Flight Times

City-to-city distance (mi)	Flight time (hr)
200	1.0
300	1.2
400	1.4
500	1.6

(1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. **(2)** Use your function to answer: **(a)** What is the time of flight if two cities are 1,000 mi apart? **(b)** What is the city-to-city distance if the flight took 2 hr? **(3)** Use your function and a spreadsheet to extend the table.

Tasks **8:1 Xavier's Notes** and **8:7 Flight Times and Distances** prominently feature linear models.

7:12

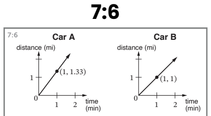
7:12 In 1972 in Loma, Montana, the temperature changed from −54°F to +49°F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

7:8

7:8 In 2018, an oil company rented an oil rig for $100,000 per day. The company drilled a well and started pumping oil. **(1)** How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for $70 in 2018. **(2)** The company estimates that the profit, P, in millions of dollars, after pumping oil for D days is $P = 0.5D - 40$. **(a)** What is the profit after the first day of pumping oil? **(b)** Make a table of pairs of values (D, P) and graph the ordered pairs. **(c)** How can the company make $30M of profit? **(3)** An equivalent expression for P is $0.5(D - 80)$. How does the 80 in this expression relate to the company's situation?

7:6

7:6



Car A and Car B were moving at constant speed, as shown in the graphs. **(1)** At the end of the first minute, how many miles had each car moved? **(2)** Which car was moving faster? **(3)** For the faster car, write a formula for the number of miles moved in n minutes. **(4)** How many miles does the faster car move in 10 minutes?

6:6

6:6 A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number of acres the farmer plants in n hours.

6:4

6:4 My car drives 570 mi with 15 gal of gas. **(1)** *Mental math/Pencil and paper* **(a)** If I drive 57 mi, I'll use ___ gal. **(b)** If I drive 5,700 mi, I'll use ___ gal. **(c)** If I have 5 gal left, I can drive ___ more mi. **(d)** I can drive ___ mi with 30 gal. **(2)** *Calculator* Calculate both unit rates for the proportional relationship. **(3)** **(a)** If I drive 532 mi, I'll use ___ gal. **(b)** If I have 11 gal left, I can drive ___ more mi. **(4)** Make a two-column table using your answers to (1a), (1c), (1d), (3a), and (3b). Then use graph paper to plot the values in the table. Label the axes of your plot.

In earlier grades, task **7:12 Temperature Change** involves a time-varying quantity that has a negative rate of change. Tasks **7:6 Car A and Car B**, **6:6 Planting Corn**, and **6:4 Gas Mileage** prominently feature proportional relationships. Task **7:8 Oil Business** involves an algebraic expression that defines a linear function.

Refer to the Standards

8.F; MP.1, MP.2, MP.4, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- Part (5) is about being aware of the limitations and hidden assumptions behind mathematical models. See the section on "How might students drive the conversation further?"

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like 8:9? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to 8:9? In what specific ways do they differ from 8:9?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like 8:9 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?