

K:13 Fluency within Five

Teacher Notes



Central math concepts

The numeracy goals for kindergarten can be summarized by listing the relevant CCSS domains and cluster headings for the grade:

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Task K:13 focuses specifically on the fluency goal for kindergarten, which is fluently adding and subtracting within 5 ([CCSS K.OA.A.5](#)). This fluency goal is best thought of not as a standalone teaching target, but rather as one expected outcome of a year's worth of extensive, cognitively rich learning experiences with the mathematics articulated by the cluster headings.

Thus, as noted in the relevant *Progression* document¹ ([p. 11](#)), "Later in the year, students solve addition and subtraction equations for numbers within 5, for example, $2 + 1 = \square$ or $3 - 1 = \square$, while still connecting these equations to situations verbally or with drawings. Experience with decompositions of numbers and with Add To and Take From situations [see the [Teacher Notes](#) for tasks **K:10 Hello, Dogs** and **K:11 Bye-Bye, Birds**] enables students to begin to fluently add and subtract within 5."

Students will generally use Level 1 methods to solve problems like those in task K:13 (see the figure, from [p. 6](#)), often using their fingers. Because using fingers is helpful for Level 2 and 3 methods in later grades, "it is important that students in kindergarten develop rapid visual and kinesthetic recognition of numbers to 5 on their fingers. Students may bring from home different ways to show numbers with their fingers and to raise (or lower) them when counting. The three major ways used around the world are starting with the thumb, the little finger, or the pointing finger (ending with the thumb in the latter two cases). Each way has advantages physically or mathematically, so students can use whatever is familiar to them. The teacher can use the range of methods present in the classroom, and these methods can be compared by students to expand their understanding of numbers. Using fingers is not a concern unless it remains at the first level of direct modeling in later grades" ([p. 8](#)).

K:13 Write or say the missing numbers.

$3 + 1 = \underline{\quad}$

$2 + 3 = \underline{\quad}$

$5 + 0 = \underline{\quad}$

$2 - 2 = \underline{\quad}$

$4 - 3 = \underline{\quad}$

$5 - 3 = \underline{\quad}$

Answer

Left column: 4, 5, 1. Right column: 5, 0, 2.

[Click here](#) for a student-facing version of the task.

Refer to the Standards

K.OA.A.5; MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency

Additional notes on the design of the task

The task allows for a spoken or written answer, at the student's discretion.

Methods used for solving single-digit addition and subtraction problems

Level 1. Direct Modeling by Counting All or Taking Away. Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Level 2. Counting On. Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).

Level 3. Convert to an Easier Problem. Decompose an addend and compose a part with another addend.

See Appendix for examples and further details.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:13? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:13? In what specific ways do they differ from K:13?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:13 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

The discussion of kindergarten in the *Progression* document ends with this guidance (p. 11): “The kindergarten standards can be stated succinctly, but they represent a great deal of focused and rich interactions in the classroom. This is necessary in order to enable all students to understand all of the numbers and concepts involved. Students who enter kindergarten without knowledge of small numbers or of counting to ten will require extra teaching time in kindergarten to meet the standards. Such time and support are vital for enabling all students to master the Grade 1 standards in Grade 1.”



Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: conceptual subitizing; understanding addition as putting together; and understanding subtraction as taking from.



Extending the task




How might students drive the conversation further?

- Students could create situations/word problems corresponding to the sums and differences in the task.
- Focusing on the problem $5 + 0 = \square$, students could look at additional examples ($2 + 0$, $6 + 0$) and state a generalized claim about the result when adding zero (CCSS MP.8).
- Focusing on the problem $2 - 2 = \square$, students could look at additional examples ($3 - 3$, $1 - 1$) and state a generalized claim about the result when subtracting a number from itself (CCSS MP.8).



Related Math Milestones tasks




K:2

K:2 There are 4  on the floor and 6  on the bed. How many  are there?

K:5

K:5 [Teacher puts 3 red counters on table.] Put some blue counters here to make 10 counters in all. [Student completes this task.] How many counters did you add? [Student determines the answer.] Write the missing number: $3 + \underline{\quad} = 10$

K:7

K:7 Hazel told a story. Write or say two numbers that will make Hazel's story true. I have 10  in my hands. I have  in my left hand. I have  in my right hand. What other numbers will also make Hazel's story true?

K:8

K:8 [Teacher holds out 5 paper clips.] How many do I have? [Student counts the paper clips.] [Teacher puts both hands behind back, then brings out 0, 1, 2, 3, 4, or 5 paper clips in one hand.] How many are in this hand? [Student counts the paper clips.] How many are in my other hand?

K:10

K:10 5  were playing. Then 3 more  came. How many  are here now?

K:11

K:11 9  were in a tree. 5  flew away. How many  are there now?

K:12


K:12 Draw 16 circles. Use a (favorite color) marker for 10 of them. Use a pencil for the rest. [Student draws.] How many are (favorite color)? How many are in pencil? Write the missing number: $16 = 10 + \underline{\quad}$

Other tasks in kindergarten that call for addition and/or subtraction are **K:2 Two Groups of Books**, **K:5 Adding to Make a Group of Ten**, **K:7 Ten Pennies, Two Hands**, **K:8 Five Behind the Back**, **K:10 Hello, Dogs**, **K:11 Bye-Bye, Birds**, and **K:12 Make Ten and Some More**.


1:9

1:9 Write the missing numbers.
 $4 + 5 = \underline{\quad}$ $7 - 4 = \underline{\quad}$
 $10 - 8 = \underline{\quad}$ $2 + 6 = \underline{\quad}$
 $4 + \underline{\quad} = 10$ $7 + \underline{\quad} = 10$

2:5

2:5 Write the value of each sum. Use as much time as you need. If you "just know it," then draw a check mark, like this: $2 + 2 = \underline{4}$ ✓ 

2:8

2:8 Write the number that makes each equation true. Use as much time as you need. 

2:3

2:3 Write the sums and differences. $36 + 45$ $72 - 17$ $64 + 27$ $82 - 55$

In later grades, task **1:9 Fluency within Ten** focuses on the fluency goal for grade 1, and tasks **2:5 Sums of Single-Digit Numbers**, **2:8 Fluency within the Addition Table**, and **2:3 Fluency within 100 (Add/Subtract)** focus on the fluency goals for grade 2.


† Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K-5, Operations and Algebraic Thinking*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.



Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions. 

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?