K:1 How Many Blocks?

Teacher Notes

Central math concepts

Several tasks in kindergarten focus directly on the domain of counting and cardinality, which is students' all-important entry point to number and operations. At a high level, counting and cardinality (how many there are) involves:[†]

- Knowing number names and the count sequence;
- Counting to tell the number of objects; and
- Comparing numbers.

Cardinal counting (counting to tell how many) is both procedural and conceptual. Cardinal counting a group of objects uses the procedure of saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. This procedure depends on students becoming fluent in saying the count sequence so that they have enough attention to focus on the pairings involved in counting objects. And conceptually, cardinal counting involves principles of cardinality:

- Understanding that the last number name said tells the number of objects counted.
- Understanding that each successive number name in the count sequence refers to a quantity that is one larger.
- Understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted.

To consider in more detail the principle that the last number name said tells the number of objects counted,[‡]

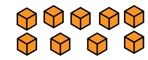
> When anyone counts, they must at the end of the counting action make a mental shift from thinking of the last counted word as referring to the last counted thing to thinking of that word as referring to all of the things (the number of things in the whole set, i.e., the cardinality of the set). For example, when counting 7 toy animals 1, 2, 3, 4, 5, 6, 7, the 7 refers to the one last animal you count when you say 7. But then you must shift to thinking of all of the animals and think of the 7 as meaning all of them: There are 7 animals. This is a major conceptual milestone for young children.

Fortunately, research shows that

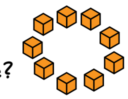
> when children discover this relationship, they tend to apply it to all counts no matter the size of the set of objects (Fuson, 1988). Therefore, this is a type of rule/principle learning that children immediately generalize and apply fairly consistently. It is relatively easy to teach children that the last word said in counting tells how many there are (see Fuson, 1988). For example, a statement of this principle followed by three demonstrations followed by another statement of the principle was sufficient to move 20 of 22 children ages 2 years 8 months to 3 years 11 months who did not use the principle to using it (Fuson, 1988).

K:1

How many blocks?
[Student tells how many.]



[Teacher slowly rearranges.]
If you count the blocks, how many do you think there will be?



Answer

First question: 9. *Second question*: 9.

Task K:1 is designed for use with manipulatives or objects. Students might also use manipulatives to support their work on other tasks.

Click here for a student-facing version of the task.

Refer to the Standards

K.CC.B.4; MP.2. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

However,

> not all children really understand cardinality, even when they understand the importance of the last counted word (Fuson, 1988). Some children initially understand only that the last word answers the "How many?" question. They do not fully grasp the more abstract idea of cardinality. Thus, they give their last counted word when asked how many there are, but they do not point to all of the objects when asked the cardinality question "Show me the seven animals." Instead, they point at the last animal again. It is important to note that responding with the last word is progress. Earlier when asked "How many are there?" children may have recounted or given a number other than the last counted word. Children who recount are understanding the question "How many are there?" as a request to count, not as a cardinal request. Such children may recount several if the question is repeated and may protest *But I already did it or I already said it* because they don't understand the reason for the repeated requests (to them, each count is a correct response to the How many are there? question).

Students who understand cardinality sometimes make errors anyway. "Counting requires effort and continued attention, and it is normal for … 5-year-olds to make occasional errors, especially on larger sets.… It is much more important for children to be enthusiastic counters who enjoy counting than for them to worry so much about errors that they are reluctant to count. … As with many physical activities, counting will improve with practice and does not need to be perfect each time" as long as all children "get frequent counting practice and watch and help each other, with occasional help and corrections from the teacher."[§]

There's a saying that runs, *It's as easy as one-two-three*. But cardinal counting isn't trivial. By posing its two sequenced questions about the 9 blocks, task K:1 offers opportunities to gain insight into a student's progress in the understandings and skills that make up the profound and educationally critical domain of counting and cardinality.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: saying the count sequence through 9; and using 1-1 correspondence.

Extending the task

How might students drive the conversation further?

- Students can respond to a prompt to "point to the 9 blocks." If students point to the whole collection, that is support for the idea that the student understands cardinality. If instead they point to the last block counted, then they may understand the importance of the last counted word but not yet grasp cardinality in full.
- Students can be asked to "make a building using 6 blocks." The response to this prompt may provide additional information about the student's development of concepts of cardinality.

Aspect(s) of rigor

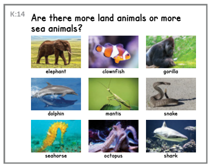
Concepts, Procedural skill and fluency

Additional notes on the design of the task

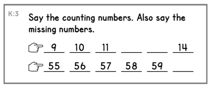
- The number of blocks is chosen to be large enough so that perceptual subitizing will not likely suffice to determine how many, but rather a counting strategy is likely to be necessary.
- In the first configuration, the blocks are grouped into 5 and 4 in case the student's solution process reveals perceptual subitizing and counting on. The second configuration is circular so as to lower the temptation to count a second time, in favor of relying on the cardinality concept to answer the question.
- The second question is hypothetical because students sometimes interpret a "how many" question not as a question about the cardinality of the collection, but rather as an instruction to perform the counting procedure itself.
- The reason for *slowly* rearranging the blocks is to help the student see that no blocks are being added or taken away during the action of rearranging.

Related Math Milestones tasks

K:14


K:14 Are there more land animals or more sea animals?

K:3

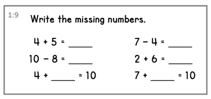

K:3 Say the counting numbers. Also say the missing numbers.

9 10 11 ___ ___ 14

55 56 57 58 59 ___

Task **K:14 Animals from Land and Sea** involves a comparison of two groups (groups which the student forms by classifying the animals). Task **K:3 Say the Numbers (Teens, Decades)** involves the counting sequence.

1:9

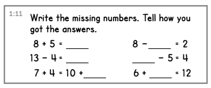

1:9 Write the missing numbers.

4 + 5 = ___ 7 − 4 = ___

10 − 8 = ___ 2 + 6 = ___

4 + ___ = 10 7 + ___ = 10

1:11

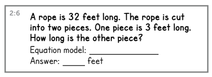

1:11 Write the missing numbers. Tell how you got the answers.

8 + 5 = ___ 8 − ___ = 2

13 − 4 = ___ ___ − 5 = 4

7 + 4 = 10 + ___ 6 + ___ = 12

2:6


2:6 A rope is 32 feet long. The rope is cut into two pieces. One piece is 3 feet long. How long is the other piece?

Equation model: ___________

Answer: ____ feet

In later grades, students use counting-on strategies and property-based strategies to solve addition and subtraction problems, as for example in tasks **1:9 Fluency within Ten** and **1:11 Using Properties and Relationships**. By grade 2, students will count, add, and subtract larger collections of items, some of which are more abstract than blocks—such as length units, for example, as in task **2:6 Cutting a Rope**.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:1? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:1? In what specific ways do they differ from K:1?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† For additional discussion, see pp. 4, 5 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

‡ Quotations are from pp. 139, 140 of *Paths Toward Excellence and Equity* (National Research Council. 2009. Washington, DC: The National Academies Press. https://doi.org/10.17226/12519).

§ Op. cit., p. 136.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

K:1 How Many Blocks?

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?