K:7 Ten Pennies, Two Hands

Teacher Notes

Math Milestones™

Central math concepts

Building on their ongoing work with counting and cardinality, students in kindergarten explore the ways in which collections of objects can be composed and decomposed, translating those patterns into relationships between numbers. This launches students on a years-long journey of developing understanding, procedural skill, and problem solving power with problems involving addition and subtraction.

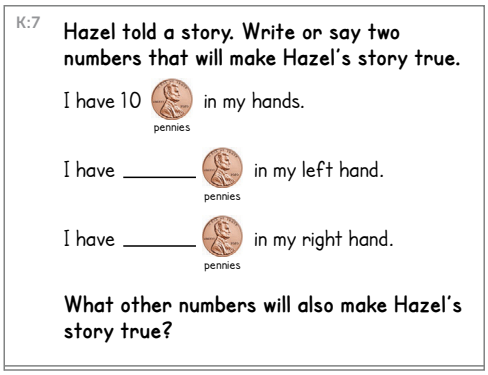
One of the important transitions in the progression from counting to adding is the transition from perceptual subitizing to conceptual subitizing. *Perceptual subitizing* is the term for when students instantly recognize and name the number of objects in a set. *Conceptual subitizing* is the term for when students use pattern recognition to quickly determine the number of objects in a set, such as seeing 2 things and 2 things and knowing this makes 4 things in all.[†] That is, conceptual subitizing involves "recognizing that a collection of objects is composed of two subcollections and quickly combining their cardinalities to find the cardinality of the collection. Use of conceptual subitizing in adding and subtracting small numbers progresses to supporting steps of more advanced methods for adding, subtracting, multiplying, and dividing single-digit numbers" in later grades.[‡]

As detailed further in the K.OA standards, kindergarten students represent addition and subtraction with objects, fingers, mental images, drawings showing the relationships among the numbers, sounds (for example, claps), acting out situations, verbal explanations, expressions, or equations. They solve addition and subtraction word problems, and add and subtract within 10, for example by using objects or drawings to represent the problem. With the benefit of extensive experiences with counting and with addition and subtraction situations, students can decompose numbers less than or equal to 10 into pairs in more than one way (for example, by using objects or drawings). Connected with these problems also is finding the number that makes 10 when added to a given number, for example by using objects or drawings.

These experiences enable students to gain grade-level fluencies and develop conceptual understandings about the operations of addition and subtraction. Across grades K–2, those conceptual understandings include the three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Word problems can be further classified according to the various possibilities for what quantities are known and what quantity is initially unknown; this leads to fifteen basic situation types for elementary addition and subtraction word problems. In particular, the situation type in task K:7 is called "Put Together/Take Apart with Both Addends Unknown."[§]



Answer

The first pair of numbers may be any two numbers in the range 0–10 that have a sum of 10. Likewise for the second pair of numbers, so long as the second pair of numbers is different from the first pair (or it could be the same pair of numbers, except with the hands switched).

Click here for a student-facing version of the task.

Task K:7 is designed for use with manipulatives or objects. Students might also use manipulatives to support their work on other tasks.

Refer to the Standards

K.OA.A.3, 4; MP.1, MP.4, MP.8.
Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Word problems or situations in which only the total is known may not be as familiar as the other kindergarten situation types ("Add To with Result Unknown," "Take From with Result Unknown," and "Put Together/Take Apart with Total Unknown"). However, the situation type "Put Together/Take Apart with Both Addends Unknown" is important in kindergarten, because by involving students in thinking about compositions and decompositions of numbers, it lays the groundwork for learning the concepts and skills of base-ten calculation in later grades—concepts and skills that rely on place value, properties of operations, and relationships between addition and subtraction. In grade 1 for example, remembering or quickly knowing the partners of 10 and the partners of single-digit numbers will be leveraged for solving single-digit addition problems using the make-ten strategy. As a specific case, to use the make-ten strategy to calculate 8 + 5, first-graders will think of the sequence of calculations

$$8 + 5 = 8 + 2 + 3 = 10 + 3 = 13.$$

The strategy requires providing the number (2) that makes 10 when added to 8, and providing the number (3) that makes 5 when added to 2.[±] The concepts and fluencies that underlie these and other grade 1 problems originate with kindergarten experiences in composing and decomposing numbers within 10, and especially finding partners of 10 as in task K:7.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: counting to tell the number of objects; and understanding addition as putting together.

Extending the task

How might students drive the conversation further?

- Students could show all decompositions of 10 and reflect on the patterns.

Related Math Milestones tasks

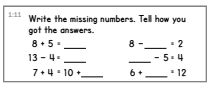
K:10

K:11

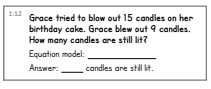
K:2

Other tasks in kindergarten that involve the kindergarten situation types are **K:10 Hello, Dogs** ("Add To with Result Unknown"), **K:11 Bye-Bye, Birds** ("Take From with Result Unknown"), and **K:2 Two Groups of Books** ("Put Together/Take Apart with Total Unknown").

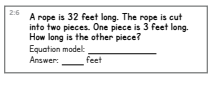
1:11



1:12



2:6



In later grades, task **1:11 Using Properties and Relationships** includes single-digit sums like 8 + 5 that cross ten. Task **1:12 Blowing Out Candles** is an unknown addend problem, and task **2:6 Cutting a Rope** is an unknown addend problem that relates addition and subtraction to length.

Additional notes on the design of the task

The task allows for a spoken or written answer, at the student's discretion.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:7? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:7? In what specific ways do they differ from K:7?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:7 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† See p. 356 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (National Research Council. 2009. Washington, DC: The National Academies Press. https://doi.org/10.17226/12519).

‡ Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona, p. 4.

§ For the other situation types, see Table 2, p. 9 of the *Progression* document.

± In the last step, the make-ten strategy also leverages student understanding of the meaning of teen numbers. (See task **K:12 Make Ten and Some More**.)

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

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Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?