K:1 How Many Blocks?

Teacher Notes

Central math concepts

Several tasks in kindergarten focus directly on the domain of counting and cardinality, which is students' all-important entry point to number and operations. At a high level, counting and cardinality (how many there are) involves:[†]

- Knowing number names and the count sequence;
- Counting to tell the number of objects; and
- Comparing numbers.

Cardinal counting (counting to tell how many) is both procedural and conceptual. Cardinal counting a group of objects uses the procedure of saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. This procedure depends on students becoming fluent in saying the count sequence so that they have enough attention to focus on the pairings involved in counting objects. And conceptually, cardinal counting involves principles of cardinality:

- Understanding that the last number name said tells the number of objects counted.
- Understanding that each successive number name in the count sequence refers to a quantity that is one larger.
- Understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted.

To consider in more detail the principle that the last number name said tells the number of objects counted,[‡]

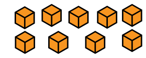
> When anyone counts, they must at the end of the counting action make a mental shift from thinking of the last counted word as referring to the last counted thing to thinking of that word as referring to all of the things (the number of things in the whole set, i.e., the cardinality of the set). For example, when counting 7 toy animals 1, 2, 3, 4, 5, 6, 7, the 7 refers to the one last animal you count when you say 7. But then you must shift to thinking of all of the animals and think of the 7 as meaning all of them: There are 7 animals. This is a major conceptual milestone for young children.

Fortunately, research shows that

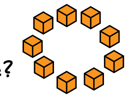
> when children discover this relationship, they tend to apply it to all counts no matter the size of the set of objects (Fuson, 1988). Therefore, this is a type of rule/principle learning that children immediately generalize and apply fairly consistently. It is relatively easy to teach children that the last word said in counting tells how many there are (see Fuson, 1988). For example, a statement of this principle followed by three demonstrations followed by another statement of the principle was sufficient to move 20 of 22 children ages 2 years 8 months to 3 years 11 months who did not use the principle to using it (Fuson, 1988).

K:1

How many blocks?
[Student tells how many.]



[Teacher slowly rearranges.]
If you count the blocks, how many do you think there will be?



Answer

First question: 9. *Second question*: 9.

Task K:1 is designed for use with manipulatives or objects. Students might also use manipulatives to support their work on other tasks.

Click here for a student-facing version of the task.

Refer to the Standards

K.CC.B.4; MP.2. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

However,

> not all children really understand cardinality, even when they understand the importance of the last counted word (Fuson, 1988). Some children initially understand only that the last word answers the "How many?" question. They do not fully grasp the more abstract idea of cardinality. Thus, they give their last counted word when asked how many there are, but they do not point to all of the objects when asked the cardinality question "Show me the seven animals." Instead, they point at the last animal again. It is important to note that responding with the last word is progress. Earlier when asked "How many are there?" children may have recounted or given a number other than the last counted word. Children who recount are understanding the question "How many are there?" as a request to count, not as a cardinal request. Such children may recount several if the question is repeated and may protest *But I already did it or I already said it* because they don't understand the reason for the repeated requests (to them, each count is a correct response to the How many are there? question).

Students who understand cardinality sometimes make errors anyway. "Counting requires effort and continued attention, and it is normal for … 5-year-olds to make occasional errors, especially on larger sets…. It is much more important for children to be enthusiastic counters who enjoy counting than for them to worry so much about errors that they are reluctant to count. … As with many physical activities, counting will improve with practice and does not need to be perfect each time" as long as all children "get frequent counting practice and watch and help each other, with occasional help and corrections from the teacher."[§]

There's a saying that runs, *It's as easy as one-two-three*. But cardinal counting isn't trivial. By posing its two sequenced questions about the 9 blocks, task K:1 offers opportunities to gain insight into a student's progress in the understandings and skills that make up the profound and educationally critical domain of counting and cardinality.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: saying the count sequence through 9; and using 1-1 correspondence.

Extending the task

How might students drive the conversation further?

- Students can respond to a prompt to "point to the 9 blocks." If students point to the whole collection, that is support for the idea that the student understands cardinality. If instead they point to the last block counted, then they may understand the importance of the last counted word but not yet grasp cardinality in full.
- Students can be asked to "make a building using 6 blocks." The response to this prompt may provide additional information about the student's development of concepts of cardinality.

Aspect(s) of rigor

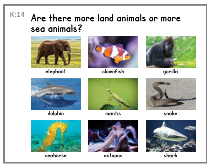
Concepts, Procedural skill and fluency

Additional notes on the design of the task

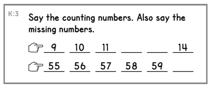
- The number of blocks is chosen to be large enough so that perceptual subitizing will not likely suffice to determine how many, but rather a counting strategy is likely to be necessary.
- In the first configuration, the blocks are grouped into 5 and 4 in case the student's solution process reveals perceptual subitizing and counting on. The second configuration is circular so as to lower the temptation to count a second time, in favor of relying on the cardinality concept to answer the question.
- The second question is hypothetical because students sometimes interpret a "how many" question not as a question about the cardinality of the collection, but rather as an instruction to perform the counting procedure itself.
- The reason for *slowly* rearranging the blocks is to help the student see that no blocks are being added or taken away during the action of rearranging.

Related Math Milestones tasks

K:14

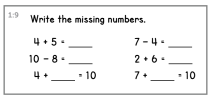


K:3

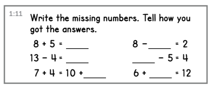


Task **K:14 Animals from Land and Sea** involves a comparison of two groups (groups which the student forms by classifying the animals). Task **K:3 Say the Numbers (Teens, Decades)** involves the counting sequence.

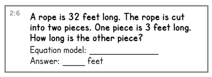
1:9



1:11



2:6



In later grades, students use counting-on strategies and property-based strategies to solve addition and subtraction problems, as for example in tasks **1:9 Fluency within Ten** and **1:11 Using Properties and Relationships**. By grade 2, students will count, add, and subtract larger collections of items, some of which are more abstract than blocks—such as length units, for example, as in task **2:6 Cutting a Rope**.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:1? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:1? In what specific ways do they differ from K:1?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:1 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† For additional discussion, see pp. 4, 5 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

‡ Quotations are from pp. 139, 140 of *Paths Toward Excellence and Equity* (National Research Council. 2009. Washington, DC: The National Academies Press. https://doi.org/10.17226/12519).

§ Op. cit., p. 136.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

K:1 How Many Blocks?

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:2 Two Groups of Books

Teacher Notes

Math Milestones™

Central math concepts

Building on their ongoing work with counting and cardinality, students in kindergarten explore the ways in which collections of objects can be composed and decomposed, joined and separated, translating those patterns into relationships between numbers. This launches students on a years-long journey of developing understanding, procedural skill, and problem solving power with problems involving addition and subtraction.

One of the important transitions in the progression from counting to adding is the transition from perceptual subitizing to conceptual subitizing. *Perceptual subitizing* is the term for when students instantly recognize and name the number of objects in a set. *Conceptual subitizing* is the term for when students use pattern recognition to quickly determine the number of objects in a set, such as seeing 2 things and 2 things and knowing this makes 4 things in all.[†] That is, conceptual subitizing involves "recognizing that a collection of objects is composed of two subcollections and quickly combining their cardinalities to find the cardinality of the collection.[‡]

Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

During grades 1 and 2, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper-elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for performing calculations with fractions are different from those for performing base-ten calculations with whole numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables. These meanings and uses begin to be learned in kindergarten.

The situation type in task K:2 is called "Put Together/Take Apart with Total Unknown."[§] Other kindergarten tasks involve the situation types of "Add To with Result Unknown," "Take From with Total Unknown," and "Put Together/Take Apart with Both Addends Unknown." Kindergarten students represent and solve these problems with objects, fingers, mental images, drawings showing the relationships among the numbers, sounds (for example, claps), acting out situations, verbal explanations, expressions,

K:2

There are 4 books on the floor

and 6 books on the bed.

How many books are there?

Answer

There are 10 books.

Click here for a student-facing version of the task.

Refer to the Standards

K.OA.A.2; MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

Since kindergarten students should see addition and subtraction equations, student work could be summarized by showing and reading to the student the equation $6 + 4 = 10$. (Student writing of equations in kindergarten is also encouraged but not expected in standards.)

or equations. These experiences enable students to gain grade-level fluencies and to develop conceptual understandings about addition and subtraction that they will draw upon in grade 1 and beyond.

Word problems vary considerably in the uses to which they put addition and subtraction, and they also vary in the complexity of the calculation required to obtain a final numerical answer. The overall challenge of a word problem depends on both the situational complexity and the computational complexity. The calculation in task K:2 involves calculating 4 + 6 or equivalently 6 + 4. Kindergarten students might calculate this total in many ways; see the gray box in the margin of page 6 of the *Progression* document and the section about "Working within 10" on pp. 10–11. Students for whom the calculation is time-consuming and/or effortful may need to be redirected to the context after obtaining the result, so as to relate the numbers in this equation to the context and answer the question in task K:2.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: counting out (or drawing) a given number of objects; counting to tell the number of objects; conceptual subitizing; and understanding addition as adding to.

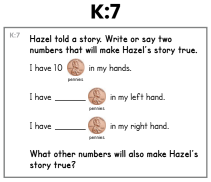
Extending the task

How might students drive the conversation further?

- Students could continue the story of the problem by supposing that 1 book was moved from the floor to the bed. What is the total number of books now?
- If some students added 4 to 6 while other students added 6 to 4, students could be shown the two equations 4 + 6 = 10 and 6 + 4 = 10 to think about and talk about why the answer was the same. Students could make statements generalizing this fact.

Related Math Milestones tasks

K:7



K:10

K:11

Other tasks in kindergarten that involve the kindergarten situation types are **K:7 Ten Pennies, Two Hands**, *Put Together/Take Apart with Both Addends Unknown*; **K:10 Hello, Dogs**, *Add To with Result Unknown*; and **K:11 Bye-Bye, Birds**, *Take From with Result Unknown*.

In later grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:2? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:2? In what specific ways do they differ from K:2?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:2 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† See p. 356 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (National Research Council. 2009. Washington, DC: The National Academies Press. https://doi.org/10.17226/12519).

‡ Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona, p. 4.

§ For the other situation types, see Table 2, p. 9 of the *Progression* document.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:3 Say the Numbers (Teens, Decades)

Teacher Notes

Central math concepts

Task K:3 combines reading and remembering: reading the given number symbols aloud as number words, and remembering the number word list for the missing portions of each sequence. The first sequence in the task deals with the passage from single-digit numbers to teen numbers, while the second sequence deals with the end of one decade and the beginning of the next.

The number words *zero, one, two, three, four, five, six, seven, eight,* and *nine* follow no mathematical system. There's also no system to the corresponding symbol sequence 0, 1, 2, 3, 4, 5, 6, 7, 8, 9—just a few suggestive hints of meaning, such as the symbol 1 consisting of a single stroke, or the symbol 3 having three "points." Then with the larger numbers 10, 11, 12, 13, 14, 15, 16, 17, 18, and 19, systematic patterns appear, patterns that continue into the decades up to the number 99. However, because of irregularities in the number word list in English, the correspondence between numerals and number words presents challenges. For example, *twenty, thirty, forty,* and *fifty* do not contain their logical root words *two, three, four*, and *five*. And the suffix -*ty* doesn't say "ten," obscuring the relationship between the decade words and the number ten that is the basis for counting the decades. These irregularities can lead to errors in reciting the number word list, such as when students say, "...thirty-eight, thirty-nine, thirty-ten, thirty-eleven, ..." Irregularities in the correspondence between number words and numerals will require additional attention in grade 1, when students learn to interpret two-digit numbers as amounts of tens and ones (see the Teacher Notes for task **1:2 Tens and Ones**).

Being able to recite the number word list in order isn't the same as understanding what quantities those words name, nor is it the same as understanding how to use the number word list as a tool for determining how many objects there are in a collection. Rather, "Students usually know or can learn to say the counting words up to a given number before they can use these numbers to count objects or to tell the number of objects." Indeed, "Students become fluent in saying the count sequence so that they have enough attention to focus on the pairings involved in counting objects" (*Progression* document,† p. 4).

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: recognizing numerals; and saying the corresponding number words.

K:3 Say the counting numbers. Also say the missing numbers.

☞ 9 10 11 ___ ___ 14

☞ 55 56 57 58 59 ___

Answer

First sequence: "Nine, ten, eleven, twelve, thirteen, fourteen." *Second sequence*: "Fifty-five, fifty-six, fifty-seven, fifty-eight, fifty-nine, sixty."

Click here for a student-facing version of the task.

Refer to the Standards

K.CC.A.1, 2; MP.6, MP.8. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency

Additional notes on the design of the task

The task isn't intended to be about intuiting a number pattern; the "pattern" here is just the standard counting sequence.

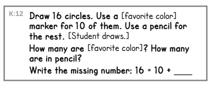
Extending the task

How might students drive the conversation further?

- The task could be repeated, with different runs of numerals each time.
- Students could write the missing numbers 12 and 13, if they have learned about writing teen numbers by that point in time (CCSS K.CC.A.3).

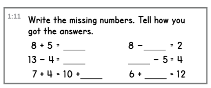
Related Math Milestones tasks

K:12



Task **K:12 Make Ten and Some More** involves a collection in which the number of objects is in the teens.

1:11



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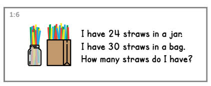

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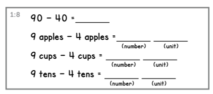

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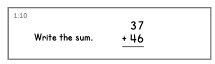


In later grades, task **1:11 Using Properties and Relationships** focuses on the extension of addition and subtraction to problems within 20. Tasks **1:6 Two Groups of Straws**, **1:8 Subtracting Units**, and **1:10 Two-Digit Addition** involve addition or subtraction with two-digit numbers.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:3? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:3? In what specific ways do they differ from K:3?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:3 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:4 Bears Talk About Shapes

Teacher Notes

Central math concepts

As explained in the *Progression* document (p. 2),[†] the three themes of elementary-grades geometry are:

- Reasoning with shape components, shape properties, and shape categories;
- Composing and decomposing shapes; and
- Spatial relations and spatial structuring.

The first two of these themes are involved in the two claims being made by the bears in task K:4.

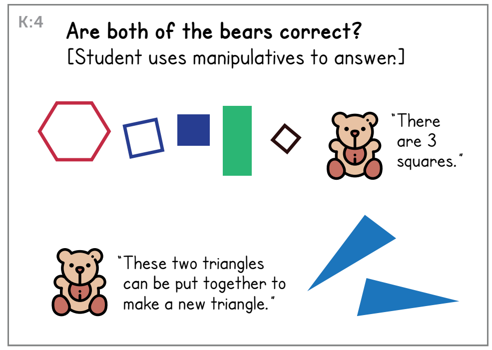
The first bear's claim involves shape categories. The *Progression* document (p. 3) describes *three levels of geometric thinking* that describe increasing sophistication with this learning progression:

- **Visual/Syncretic level**. Students recognize shapes, for example, a rectangle "looks like a door."
- **Descriptive level**. Students perceive properties of shapes, for example, a rectangle has four sides, all its sides are straight, opposite sides have equal length.
- **Analytic level**. Students characterize shapes by their properties, for example, a rectangle has opposite sides of equal length and four right angles.
- **Abstract level**. Students understand, for example, that a rectangle is a parallelogram because it has all the properties of parallelograms.

The first bear's claim can be settled using Visual/Syncretic and/or Descriptive reasoning. The task provides experience with identifying shapes regardless of non-essential attributes such as color, size, and orientation (p. 6).

To evaluate the second bear's claim, students can join the two triangles to make a new triangle, as shown in the figure that appears in the "Answer" section. Kindergarten students "not only build shapes from components, but also compose shapes to build pictures and designs. Initially lacking competence in composing geometric shapes, they gain abilities to combine shapes—first by trial and error and gradually by considering components—into pictures. At first, side length is the only component considered. Later experience brings an intuitive appreciation of angle size" (p. 7).

The fact that two distinct triangles can be created by joining the given triangles is an example of the general fact that many problems in mathematics have more than one solution. For another example in this grade, see task **K:7 Ten Pennies, Two Hands**.



Answer

Both bears are correct. With respect to the second bear's claim, the figure shows two possible ways to make a new triangle by putting together the two given triangles.

Task K:4 is designed for use with manipulatives or objects. Students might also use manipulatives to support their work on other tasks.

Click here for a student-facing version of the task.

Refer to the Standards

K.G.A.2, K.G.B.4,6; MP.1, MP.3, MP.5, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: counting to tell how many; perceptual subitizing; and recognizing and naming shapes.

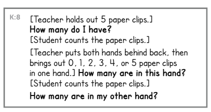
Extending the task

How might students drive the conversation further?

- With reference to the first bear's claim, if students correctly indicate that the bear is correct, they could continue the discussion by identifying the squares and counting them.
- With reference to the second bear's claim, students who composed a new triangle by joining the given triangles along the longer legs can compare their shape with the shape of a partner who composed a new triangle by joining the given triangles along the shorter legs. The two partners can then consider the problem of adding a rectangle to their two new triangles in such a way as to make a sailboat design.

Related Math Milestones tasks

K:8

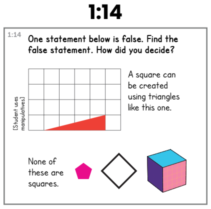

K:8 [Teacher holds out 5 paper clips.]
How many do I have?
[Student counts the paper clips.]
[Teacher puts both hands behind back, then brings out 0, 1, 2, 3, 4, or 5 paper clips in one hand.] **How many are in this hand?**
[Student counts the paper clips.]
How many are in my other hand?

K:10

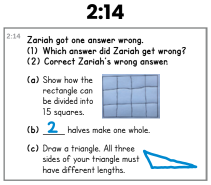

K:10 5 dogs were playing.
Then 3 more dogs came.
How many dogs are here now?

Counting, subitizing, or counting out small collections of 5 or fewer objects may be part of many kindergarten tasks, such as **K:8 Five Behind the Back**, **K:10 Hello, Dogs**, and others.

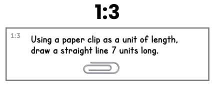
1:14


1:14 One statement below is false. Find the false statement. How did you decide?
[Student uses manipulatives]
A square can be created using triangles like this one.
None of these are squares.

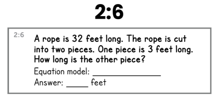
2:14


2:14 Zariah got one answer wrong.
(1) Which answer did Zariah get wrong?
(2) Correct Zariah's wrong answer.
(a) Show how the rectangle can be divided into 15 squares.
(b) 2 halves make one whole.
(c) Draw a triangle. All three sides of your triangle must have different lengths.

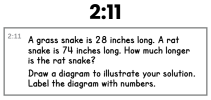
1:3


1:3 Using a paper clip as a unit of length, draw a straight line 7 units long.

2:6


2:6 A rope is 32 feet long. The rope is cut into two pieces. One piece is 3 feet long. How long is the other piece?
Equation model: ______________
Answer: _____ feet

2:11


2:11 A grass snake is 28 inches long. A rat snake is 74 inches long. How much longer is the rat snake?
Draw a diagram to illustrate your solution. Label the diagram with numbers.

In later grades, tasks **1:14 Shape True/False** and **2:14 Correcting a Shape Answer** involve the three themes of elementary-grades geometry listed under "Central math concepts" for task K:4. Spatial structuring and composing/decomposing are involved in length measurement (iterating length units), as in task **1:3 Paper Clip Length Units**. Composing and decomposing is involved in adding and subtracting lengths, as in tasks **2:6 Cutting a Rope** and **2:11 Grass Snake vs. Rat Snake**.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

The second bear's claim is intended to be explored using manipulatives. The first bear's claim could also be explored using manipulatives.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:4? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:4? In what specific ways do they differ from K:4?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:4 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common Core State Standards in Mathematics (draft). Grades K–5, Geometry*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona. Page numbers in these Teacher Notes refer to this *Progression*.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:5 Adding to Make a Group of Ten

Teacher Notes

Central math concepts

Building on their ongoing work with counting and cardinality, students in kindergarten explore the ways in which collections of objects can be composed and decomposed, translating those patterns into relationships between numbers. This launches students on a years-long journey of developing understanding, procedural skill, and problem solving power with problems involving addition and subtraction.[†] In addition to solving contextual problems involving addition and subtraction,[‡] students also explore number relationships, especially when decomposing numbers 1–10 into pairs in more than one way (CCSS K.OA.A.4) and finding the partners of ten (CCSS K.OA.A.5). Finding partners of ten means that for any number 1–9, students find the number that makes 10 when added to the given number. Mathematically, this problem has an unknown-addend structure. Students might find the unknown addend using objects or drawings, and they record the answer with a drawing or, as in task K:5, an equation.

Part of task K:5 involves writing a numeral. Kindergarten students write numbers from 0 to 20 and represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects) (CCSS K.CC.A.3). As explained in *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*,[§]

> Learning to write number symbols (numerals) is a much more difficult task than is reading them and often is not begun until kindergarten. Writing numerals requires children to have an accurate mental image of the symbol, which entails left-right orientation, and a motor plan to translate the mental image into the correct sequence of motor actions to form a numeral.... Some numerals are much easier than others. The loops in 6 and 9, the curve and straight line in the 2, and the crossovers in the 8 are difficult but can be mastered by kindergarten children with effort. The easier numerals 1, 3, 4, 5, and 7 can often be mastered earlier. Whenever children do learn to write numerals, learning to write correct and readable numerals is not enough. They must become fluent at writing numerals (i.e., writing numerals must become overlearned) so that writing them as part of a more complex task is not so slow or effortful as to be discouraging when solving several problems. It is common for children at this step and even later to reverse some numerals (such as 3) because the left-right orientation is difficult for them. This will become easier with age and experience. (p. 138)

Finding partners of 10 in kindergarten is important preparation for strategies such as making ten in grade 1 that allow students to extend addition and subtraction beyond the single digits to problems within 20. For example, to add 7 + 8, students in grade 1 can think that the partner of 10 for 7 is 3, and for this reason decompose 8 as 3 + 5, which changes the problem 7 + 8 into the easier problem (7 + 3) + 5 or 10 + 5, which (based on an understanding of the teen numbers) equals 15.

K:5 [Teacher puts 3 red counters on table.] **Put some blue counters here to make 10 counters in all.** [Student completes this task.] **How many counters did you add?** [Student determines the answer.]

Write the missing number: 3 + ____ = 10

Answer

7.

Click here for a student-facing version of the task.

Task K:5 is designed for use with manipulatives or objects. Students might also use manipulatives to support their work on other tasks.

Refer to the Standards

K.OA.A.4; MP.1, MP.2, MP.5, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: cardinal counting; counting out a collection; conceptual subitizing; saying the counting sequence through 10; and concepts of addition and subtraction.

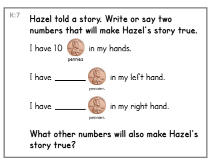
Extending the task

How might students drive the conversation further?

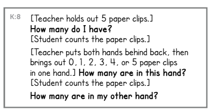
- The task could be repeated, with different known addends each time.
- Students could pose the task to the teacher, a partner, or a caregiver or family member.
- Students could show all decompositions of 10 and reflect on the patterns.

Related Math Milestones tasks

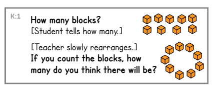
K:7


K:7 **Hazel told a story. Write or say two numbers that will make Hazel's story true.**
I have 10 pennies in my hands.
I have ______ pennies in my left hand.
I have ______ pennies in my right hand.
What other numbers will also make Hazel's story true?

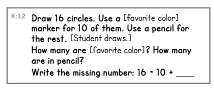
K:8


K:8 [Teacher holds out 5 paper clips.]
How many do I have?
[Student counts the paper clips.]
[Teacher puts both hands behind back, then brings out 0, 1, 2, 3, 4, or 5 paper clips in one hand.] **How many are in this hand?**
[Student counts the paper clips.]
How many are in my other hand?

K:1

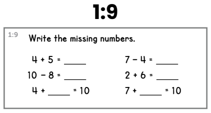

K:1 **How many blocks?**
[Student tells how many.]
[Teacher slowly rearranges.]
If you count the blocks, how many do you think there will be?

K:12

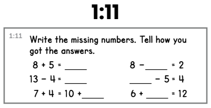

K:12 **Draw 16 circles. Use a** [favorite color] **marker for 10 of them. Use a pencil for the rest.** [Student draws.]
How many are [favorite color]**? How many are in pencil?**
Write the missing number: 16 = 10 + ____

Task **K:7 Ten Pennies, Two Hands** involves decomposing 10 into pairs in more than one way. Task **K:8 Five Behind the Back** has an unknown-addend structure. **K:1 How Many Blocks?** involves counting and cardinality. Task **K:12 Make Ten and Some More** involves a decomposition of 16.

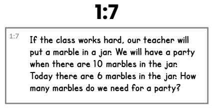
1:9


1:9 Write the missing numbers.
4 + 5 = ____ 7 − 4 = ____
10 − 8 = ____ 2 + 6 = ____
4 + ____ = 10 7 + ____ = 10

1:11


1:11 Write the missing numbers. Tell how you got the answers.
8 + 5 = ____ 8 − ____ = 2
13 − 4 = ____ ____ − 5 = 4
7 + 4 = 10 + ____ 6 + ____ = 12

1:7


1:7 If the class works hard, our teacher will put a marble in a jar. We will have a party when there are 10 marbles in the jar. Today there are 6 marbles in the jar. How many marbles do we need for a party?

In later grades, partners of 10 are part of task **1:9 Fluency within Ten**, and partners of 10 play a role in strategies useful for the problems in task **1:11 Using Properties and Relationships**. Task **1:7 Class Marble Jar** involves partners of 10 in context, in a situation of "Add To with Change Unknown." (See the Map of Addition and Subtraction Situations in K–2 Math Milestones.)

Additional notes on the design of the task

- The equation 3 + 7 = 10 in the task reflects a mental process of joining red counters and blue counters into a total group of—simply—counters. The choice of unit (what gets counted) is thus central in the task. The task would make less sense, for example, if the 3 red counters and 7 blue counters had been instead 3 red counters and 7 cups of water.
- As a warm-up question, the student could initially be asked, "How many red counters are there?" before being asked to put some blue counters here to make 10 counters in all.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:5? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:5? In what specific ways do they differ from K:5?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:5 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† For additional discussion, see pp. 4–11 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

‡ See the Map of Addition and Subtraction Situations in K–2 Math Milestones.

§ National Research Council. (2009). *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*. Committee on Early Childhood Mathematics, Christopher T. Cross, Taniesha A. Woods, and Heidi Schweingruber, Editors. Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: The National Academies Press.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:6 More Shells or More Stars?

Teacher Notes

Central math concepts

Several tasks in kindergarten focus directly on the domain of counting and cardinality, which is students' all-important entry point to number and operations. At a high level, counting and cardinality (how many there are) involves:†

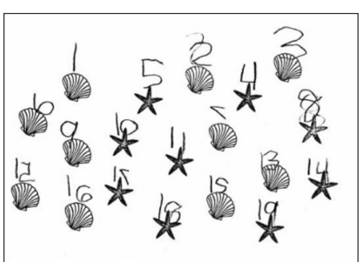
- Knowing number names and the count sequence;
- Counting to tell the number of objects; and
- Comparing numbers.

Cardinal counting (counting to tell how many) is both procedural and conceptual. Cardinal counting a group of objects uses the procedure of saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. This procedure depends on students becoming fluent in saying the count sequence, so that they have enough attention to focus on the pairings involved in counting objects. And conceptually, cardinal counting involves principles of cardinality:

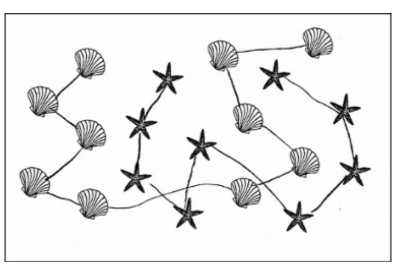
- Understanding that the last number name said tells the number of objects counted.
- Understanding that each successive number name in the count sequence refers to a quantity that is one larger.
- Understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted.

When it comes to comparing numbers, the focus in kindergarten is on comparing the amounts in two collections of objects. Students "[i]dentify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, for example, by using matching and counting strategies" (CCSS K.CC.C.6).

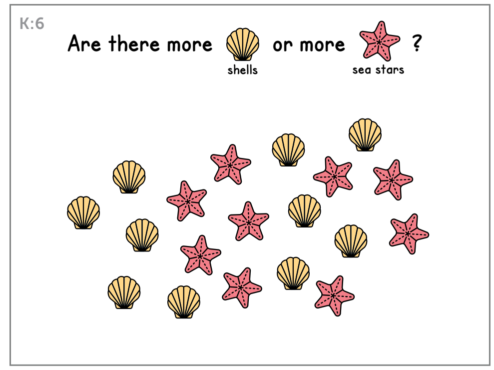
In task K:6 in particular, comparing the number of shells to the number of sea stars involves conceiving of the total group of objects as two groups, and using a method to keep track of the counts of two scattered arrangements (see the two samples of student work).



A student counted the total group, instead of counting two groups.



For each group, a student drew a line to keep track of the objects counted.



Answer

There are more shells. (Students should use counting to decide, rather than making a guess or an unsupported claim.)

Click here for a student-facing version of the task.

Refer to the Standards

K.CC.B.5; MP.1, MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: saying the count sequence through 10; counting with 1-1 correspondence; and concepts of cardinality.

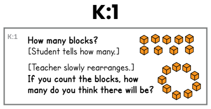
Extending the task

How might students drive the conversation further?

- Two students who got different answers could show each other how they determined the answer and resolve their disagreement.
- Students who can count out objects with support can create a "puzzle" version of task K:6 using numbers that are within their known count sequence and for which they have cardinal-counting experience and experience comparing groups of objects, by drawing two collections of objects (or assembling objects, or pasting cutouts to a sheet of paper, etc.). Students could ask a caregiver, parent, or family member to solve their puzzle.

Related Math Milestones tasks

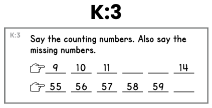
K:1


K:1 How many blocks?
[Student tells how many.]
[Teacher slowly rearranges.]
If you count the blocks, how many do you think there will be?

K:14

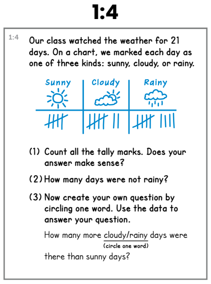

K:14 Are there more land animals or more sea animals?

K:3


K:3 Say the counting numbers. Also say the missing numbers.
9 10 11 ___ ___ 14
55 56 57 58 59 ___

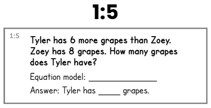
Task **K:1 How Many Blocks?** involves concepts and skills of counting and cardinality. Task **K:14 Animals from Land and Sea** involves a comparison of two groups (groups which the student forms by classifying the animals). Task **K:3 Say the Numbers (Teens, Decades)** involves the counting sequence.

1:4


1:4 Our class watched the weather for 21 days. On a chart, we marked each day as one of three kinds: sunny, cloudy, or rainy.

(1) Count all the tally marks. Does your answer make sense?
(2) How many days were not rainy?
(3) Now create your own question by circling one word. Use the data to answer your question.
How many more cloudy/rainy (circle one word) days were there than sunny days?

1:5


1:5 Tyler has 6 more grapes than Zoey. Zoey has 8 grapes. How many grapes does Tyler have?
Equation model: ____________
Answer: Tyler has _____ grapes.

In later grades, students begin to quantify how many more are in one group than another, as in the word problems in tasks **1:4 Analyzing Weather Data** (part (3), situation type "Compare with Difference Unknown") and **1:5 Tyler's Grapes** (situation type "Compare with Bigger Unknown"). For Compare problems in grade 2, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

Additional notes on the design of the task

- Observing a student while they work on the task can provide information about development in the progression of skills and concepts in counting and cardinality.
- Completing the task doesn't require students to write a comparison statement such as 10 > 9 because writing comparisons that include the symbols > and < is introduced in grade 1.
- Students might know about sea stars under the name *starfish*.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:6? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:6? In what specific ways do they differ from K:6?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:6 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† For additional discussion, see pp. 4, 5 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

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K:6 More Shells or More Stars?

Teacher Notes

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
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Language

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Identity, Agency, and Belonging

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- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:7 Ten Pennies, Two Hands

Teacher Notes

Central math concepts

Building on their ongoing work with counting and cardinality, students in kindergarten explore the ways in which collections of objects can be composed and decomposed, translating those patterns into relationships between numbers. This launches students on a years-long journey of developing understanding, procedural skill, and problem solving power with problems involving addition and subtraction.

One of the important transitions in the progression from counting to adding is the transition from perceptual subitizing to conceptual subitizing. *Perceptual subitizing* is the term for when students instantly recognize and name the number of objects in a set. *Conceptual subitizing* is the term for when students use pattern recognition to quickly determine the number of objects in a set, such as seeing 2 things and 2 things and knowing this makes 4 things in all.[†] That is, conceptual subitizing involves "recognizing that a collection of objects is composed of two subcollections and quickly combining their cardinalities to find the cardinality of the collection. Use of conceptual subitizing in adding and subtracting small numbers progresses to supporting steps of more advanced methods for adding, subtracting, multiplying, and dividing single-digit numbers" in later grades.[‡]

As detailed further in the K.OA standards, kindergarten students represent addition and subtraction with objects, fingers, mental images, drawings showing the relationships among the numbers, sounds (for example, claps), acting out situations, verbal explanations, expressions, or equations. They solve addition and subtraction word problems, and add and subtract within 10, for example by using objects or drawings to represent the problem. With the benefit of extensive experiences with counting and with addition and subtraction situations, students can decompose numbers less than or equal to 10 into pairs in more than one way (for example, by using objects or drawings). Connected with these problems also is finding the number that makes 10 when added to a given number, for example by using objects or drawings.

These experiences enable students to gain grade-level fluencies and develop conceptual understandings about the operations of addition and subtraction. Across grades K–2, those conceptual understandings include the three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Word problems can be further classified according to the various possibilities for what quantities are known and what quantity is initially unknown; this leads to fifteen basic situation types for elementary addition and subtraction word problems. In particular, the situation type in task K:7 is called "Put Together/Take Apart with Both Addends Unknown."[§]

K:7 Hazel told a story. Write or say two numbers that will make Hazel's story true.

I have 10 pennies in my hands.

I have ______ pennies in my left hand.

I have ______ pennies in my right hand.

What other numbers will also make Hazel's story true?

Answer

The first pair of numbers may be any two numbers in the range 0–10 that have a sum of 10. Likewise for the second pair of numbers, so long as the second pair of numbers is different from the first pair (or it could be the same pair of numbers, except with the hands switched).

Click here for a student-facing version of the task.

Task K:7 is designed for use with manipulatives or objects. Students might also use manipulatives to support their work on other tasks.

Refer to the Standards

K.OA.A.3, 4; MP.1, MP.4, MP.8.
Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

Word problems or situations in which only the total is known may not be as familiar as the other kindergarten situation types ("Add To with Result Unknown," "Take From with Result Unknown," and "Put Together/Take Apart with Total Unknown"). However, the situation type "Put Together/Take Apart with Both Addends Unknown" is important in kindergarten, because by involving students in thinking about compositions and decompositions of numbers, it lays the groundwork for learning the concepts and skills of base-ten calculation in later grades—concepts and skills that rely on place value, properties of operations, and relationships between addition and subtraction. In grade 1 for example, remembering or quickly knowing the partners of 10 and the partners of single-digit numbers will be leveraged for solving single-digit addition problems using the make-ten strategy. As a specific case, to use the make-ten strategy to calculate 8 + 5, first-graders will think of the sequence of calculations

$$8 + 5 = 8 + 2 + 3 = 10 + 3 = 13.$$

The strategy requires providing the number (2) that makes 10 when added to 8, and providing the number (3) that makes 5 when added to 2.± The concepts and fluencies that underlie these and other grade 1 problems originate with kindergarten experiences in composing and decomposing numbers within 10, and especially finding partners of 10 as in task K:7.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: counting to tell the number of objects; and understanding addition as putting together.

Extending the task

How might students drive the conversation further?

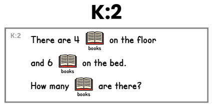
- Students could show all decompositions of 10 and reflect on the patterns.

Related Math Milestones tasks

K:10

K:11

K:2



Other tasks in kindergarten that involve the kindergarten situation types are **K:10 Hello, Dogs** ("Add To with Result Unknown"), **K:11 Bye-Bye, Birds** ("Take From with Result Unknown"), and **K:2 Two Groups of Books** ("Put Together/Take Apart with Total Unknown").

1:11

1:11 Write the missing numbers. Tell how you got the answers.

8 + 5 = ____ 8 − ____ = 2
13 − 4 = ____ ____ − 5 = 4
7 + 4 = 10 + ____ 6 + ____ = 12

1:12

1:12 Grace tried to blow out 15 candles on her birthday cake. Grace blew out 9 candles. How many candles are still lit?
Equation model: ____________
Answer: _____ candles are still lit.

2:6

2:6 A rope is 32 feet long. The rope is cut into two pieces. One piece is 3 feet long. How long is the other piece?
Equation model: ____________
Answer: _____ feet

In later grades, task **1:11 Using Properties and Relationships** includes single-digit sums like 8 + 5 that cross ten. Task **1:12 Blowing Out Candles** is an unknown addend problem, and task **2:6 Cutting a Rope** is an unknown addend problem that relates addition and subtraction to length.

Additional notes on the design of the task

The task allows for a spoken or written answer, at the student's discretion.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:7? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:7? In what specific ways do they differ from K:7?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:7 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† See p. 356 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (National Research Council. 2009. Washington, DC: The National Academies Press. https://doi.org/10.17226/12519).

‡ Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona, p. 4.

§ For the other situation types, see Table 2, p. 9 of the *Progression* document.

± In the last step, the make-ten strategy also leverages student understanding of the meaning of teen numbers. (See task **K:12 Make Ten and Some More.**)

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:8 Five Behind the Back

Teacher Notes

Central math concepts

In task K:8, the student answers a succession of three questions. The first and second questions are "how-many" questions—that is, questions about cardinality (CCSS K.CC). Cardinal counting (counting to tell how many) is both procedural and conceptual. Cardinal counting a group of objects uses the procedure of saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. This procedure depends on students becoming fluent in saying the count sequence, so that they have enough attention to focus on the pairings involved in counting objects. And conceptually, cardinal counting involves principles of cardinality:

- Understanding that the last number name said tells the number of objects counted.
- Understanding that each successive number name in the count sequence refers to a quantity that is one larger.
- Understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted.

Students might answer the two "how-many" questions in task K:8 by counting or (given the small numbers involved) by perceptual or conceptual subitizing. *Perceptual subitizing* is the term for when students instantly recognize and name the number of objects in a set. *Conceptual subitizing* is the term for when students use pattern recognition to quickly determine the number of objects in a set, such as seeing 2 things and 2 things and knowing this makes 4 things in all.[†]

As for the third question in task K:8, mathematically it has the structure of an unknown-addend problem—for example, 4 + ☐ = 5 in a case where 4 paper clips are shown. Kindergarten students don't routinely work with unknown-addend word problems, but kindergarten students do decompose numbers such as 5 into pairs in more than one way, for example, by using objects or drawings (CCSS K.OA.A.3). More generally, building on their ongoing work with counting and cardinality, students in kindergarten explore the ways in which collections of objects can be composed and decomposed, translating those patterns into relationships between numbers. This launches students on a years-long journey of developing understanding, procedural skill, and problem solving power with problems involving addition and subtraction.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: perceptual subitizing; conceptual subitizing; saying the counting sequence through 5; cardinal counting; and concepts of addition and subtraction.

K:8 [Teacher holds out 5 paper clips.]
How many do I have?
[Student counts the paper clips.]
[Teacher puts both hands behind back, then brings out 0, 1, 2, 3, 4, or 5 paper clips in one hand.] **How many are in this hand?**
[Student counts the paper clips.]
How many are in my other hand?

Answer

First question: 5. *Second question*: Answers vary depending on the number of paper clips shown. *Third question*: Answers vary depending on the number of paper clips shown, but the sum of the answer and the number of paper clips shown must be 5.

Click here for a student-facing version of the task.

Task K:8 is designed for use with manipulatives or objects. Students might also use manipulatives to support their work on other tasks.

Refer to the Standards

K.OA.A; MP.1, MP.2. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

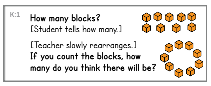
Extending the task

How might students drive the conversation further?

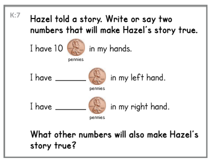
- The task could be repeated, with different numbers of paper clips shown in one hand each time.
- Students could pose the task to the teacher, a partner, or a caregiver or family member.
- Students could show all decompositions of 5 and reflect on the patterns.

Related Math Milestones tasks

K:1


K:1 How many blocks?
[Student tells how many.]
[Teacher slowly rearranges.]
If you count the blocks, how many do you think there will be?

K:7


K:7 Hazel told a story. Write or say two numbers that will make Hazel's story true.

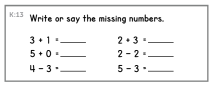
I have 10 pennies in my hands.

I have ______ pennies in my left hand.

I have ______ pennies in my right hand.

What other numbers will also make Hazel's story true?

K:13


K:13 Write or say the missing numbers.

3 + 1 = ______ 2 + 3 = ______
5 + 0 = ______ 2 − 2 = ______
4 − 3 = ______ 5 − 3 = ______

Task **K:1 How Many Blocks?** involves counting and cardinality. Task **K:7 Ten Pennies, Two Hands** has the situation type "Put Together/Take Apart with Both Addends Unknown." Task **K:13 Fluency within Five** focuses on the fluency goal for kindergarten.

In later grades, see the [Map of Addition and Subtraction Situations in K–2 Math Milestones]().

Additional notes on the design of the task

- Because the total number of paper clips (5) matches the fluency goal for kindergarten (CCSS K.OA.C.5; see the Teacher Notes for task **K:13 Fluency within Five**), task K:8 can provide teachers with information about students' development towards fluency.
- The task refers to paper clips, but other small objects could be used.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:8? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:8? In what specific ways do they differ from K:8?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:8 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† See p. 356 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (National Research Council. 2009. Washington, DC: The National Academies Press. https://doi.org/10.17226/12519).

‡ Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:9 Compare 6 and 5

Teacher Notes

Central math concepts

Several tasks in kindergarten focus directly on the domain of counting and cardinality, which is students' all-important entry point to number and operations. At a high level, counting and cardinality (how many there are) involves:†

- Knowing number names and the count sequence;
- Counting to tell the number of objects; and
- Comparing numbers.

The learning in this progression is a blend of fluency and conceptual learning. Consider counting to tell the number of objects—sometimes called "cardinal counting." Cardinal counting is both procedural and conceptual. Cardinal counting a group of objects uses the procedure of saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. This procedure depends on students becoming fluent in saying the count sequence, so that they have enough attention to focus on the pairings involved in counting objects. And conceptually, cardinal counting involves principles of cardinality:

- Understanding that the last number name said tells the number of objects counted.
- Understanding that each successive number name in the count sequence refers to a quantity that is one larger.
- Understanding that the number of objects is the same regardless of their arrangement or the order in which they were counted.

As for comparing numbers, the focus in kindergarten is on comparing the amounts in two collections of objects. Students "Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies" (CCSS K.CC.C.6). After extensive experiences of this kind, students compare two numbers between 1 and 10 presented as written numerals. Listening to students as they tell how they decided which number was greater can provide information about their knowledge and skill in the counting and cardinality domain, such as recognizing written numerals, being able to count forward beginning from a given number within the known sequence, or understanding that each successive number name refers to a quantity that is one larger.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: recognizing written numbers; saying the count sequence; counting forward beginning from 5; relating the symbols 5 and 6 to the cardinalities they refer to; and communicating mathematical reasoning about numbers.

K:9

6 5

Point to the greater number. [Student points.] **Tell me how you decided.**

Answer

Pointing to 6 or communicating in some other way that 6 is the greater number. Explanations may vary but could involve: discussing the quantities involved (as in, "If you have six things, you have more than five"); counting out or drawing a group of 5 objects and comparing it to (or continuing to build) a group of 6 objects; the addition idea that 5 + 1 = 6 (you have to add 1 to 5 to make 6, so 6 must be more than 5); the subtraction idea that 6 - 1 = 5 (if we take 1 away from 6 we get 5, then 6 must be more than 5); observing that we say "six" after "five" in the count sequence; or other approaches.

Click here for a student-facing version of the task.

Refer to the Standards

K.CC.B.4c, K.CC.C.7; MP.2, MP.3. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Procedural skill and fluency

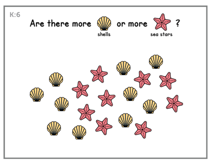
Extending the task

How might students drive the conversation further?

- Two students who gave different explanations could listen to each other's explanations, and each could tell each other if they understood.
- Students could write their own version of task K:9 using numbers that are within their known count sequence and for which they have cardinal-counting experience and experience comparing groups of objects.
- Students who can count out 5–6 objects with support could turn task K:9 into a "puzzle" analogous to task **K:6 More Shells or More Stars?** by drawing two collections of objects (or pasting objects to a sheet of paper, etc.). Students could ask a parent or caregiver to solve their puzzle.

Related Math Milestones tasks

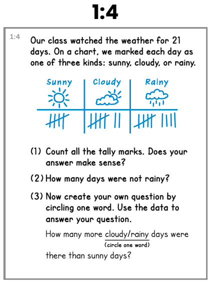
K:6



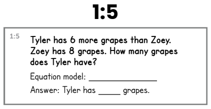
K:14

Task **K:6 More Shells or More Stars?** involves procedural fluency in the use of counting to determine which of two groups has more objects. Task **K:14 Animals from Land and Sea** involves a comparison of two groups (groups which the student forms by classifying the animals).

1:4



1:5



In later grades, task **1:4 Analyzing Weather Data** (part (3)) involves the situation type "Compare with Difference Unknown," in which students compare two amounts in context by calculating the difference between them. Task **1:5 Tyler's Grapes** involves the situation type "Compare with Bigger Unknown," in which students use information about a comparison in context to deduce the bigger of two numbers being compared.

Additional notes on the design of the task

Completing the task doesn't require students to write a comparison statement such as 6 > 5, because writing comparisons that include the symbols > and < is introduced in grade 1.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:9? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:9? In what specific ways do they differ from K:9?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:9 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† For additional discussion, see pp. 4, 5 of *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking* (Common Core Standards Writing Team, May 29, 2011. Tucson, AZ: Institute for Mathematics and Education, University of Arizona).

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Anticipating and responding to student thinking about the task

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On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:10 Hello, Dogs

Teacher Notes

Central math concepts

Building on their ongoing work with counting and cardinality, students in kindergarten explore the ways in which collections of objects can be composed and decomposed, translating those patterns into relationships between numbers. This launches students on a years-long journey of developing understanding, procedural skill, and problem solving power with problems involving addition and subtraction.

One of the important transitions in the progression from counting to adding is the transition from perceptual subitizing to conceptual subitizing. *Perceptual subitizing* is the term for when students instantly recognize and name the number of objects in a set. *Conceptual subitizing* is the term for when students use pattern recognition to quickly determine the number of objects in a set, such as seeing 2 things and 2 things and knowing this makes 4 things in all.[†] That is, conceptual subitizing involves "recognizing that a collection of objects is composed of two subcollections and quickly combining their cardinalities to find the cardinality of the collection.[‡]

Across grades K–2, students solve problems involving three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Elementary word problems in addition and subtraction can be classified as belonging to one of these three main kinds. Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

During grades 1 and 2, students work with all situation types and all variations in the known and unknown quantities, with quantities given as whole numbers. In the upper elementary grades, these understandings of addition and subtraction are applied and extended to solve problems involving fractional quantities. Although the algorithms for performing calculations with fractions are different from those for performing base-ten calculations with whole numbers, the underlying meanings and uses of addition and subtraction are the same regardless of whether the numbers involved are whole numbers, fractions, decimals, or even variables. These meanings and uses begin to be learned in kindergarten.

The situation type in task K:10 is called "Add To with Result Unknown." Other kindergarten tasks involve the situation types of "Add To with Result Unknown," "Put Together/Take Apart with Total Unknown,"[§] and "Put Together/Take Apart with Both Addends Unknown." Kindergarten students represent and solve these problems with objects, fingers, mental images, drawings showing the relationships among the numbers, sounds (for example, claps), acting out situations, verbal explanations, expressions,

Answer

8 dogs are here now.

Click here for a student-facing version of the task.

Refer to the Standards

K.OA.A.2; MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

Since kindergarten students should see addition and subtraction equations, student work could be summarized by showing and reading to the student the equation 5 + 3 = 8. (Student writing of equations in kindergarten is also encouraged but not expected in standards.)

or equations. These experiences enable students to gain grade-level fluencies and to develop conceptual understandings about addition and subtraction that they will draw upon in grade 1 and beyond.

Word problems vary considerably in the uses to which they put addition and subtraction, and they also vary in the complexity of the calculation required to obtain a final numerical answer. The overall challenge of a word problem depends on both the situational complexity and the computational complexity. The calculation in task K:10 involves calculating 5 + 3. Kindergarten students might calculate this total in many ways; see the gray box in the margin of page 6 of the *Progression* document and the section about "Working within 10" on pages 10–11. Students for whom the calculation 5 + 3 is time-consuming and/or effortful may need to be redirected to the context after obtaining the result 5 + 3 = 8, so as to relate the numbers in this equation to the context and answer the question in task K:10.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: counting out (or drawing) a given number of objects; counting to tell the number of objects; conceptual subitizing; and understanding addition as adding to.

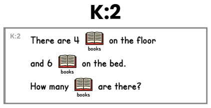
Extending the task

How might students drive the conversation further?

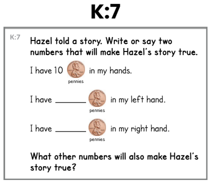
- Students could continue the story of the problem by supposing that a chosen number of the dogs decided to leave. After that, how many dogs are here?
- What if 8 dogs decided to leave—how many dogs would be here then?

Related Math Milestones tasks

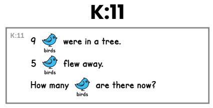
K:2



K:7



K:11



Other tasks in kindergarten that involve the kindergarten situation types are **K:2 Two Groups of Books**, *Put Together/Take Apart with Total Unknown*; **K:7 Ten Pennies, Two Hands**, *Put Together/Take Apart with Both Addends Unknown*; and **K:11 Bye-Bye, Birds**, *Take From with Result Unknown*.

In later grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:10? Locate 2–3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:10? In what specific ways do they differ from K:10?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:10 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† See p. 356 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (National Research Council. 2009. Washington, DC: The National Academies Press. https://doi.org/10.17226/12519).

‡ Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona, p. 4.

§ For the other situation types, see Table 2, p. 9 of the *Progression* document.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:11 Bye-Bye, Birds

Teacher Notes

Central math concepts

Building on their ongoing work with counting and cardinality, students in kindergarten explore the ways in which collections of objects can be composed and decomposed, translating those patterns into relationships between numbers. This launches students on a years-long journey of developing understanding, procedural skill, and problem solving power with problems involving addition and subtraction.

One of the important transitions in the progression from counting to adding is the transition from perceptual subitizing to conceptual subitizing. *Perceptual subitizing* is the term for when students instantly recognize and name the number of objects in a set. *Conceptual subitizing* is the term for when students use pattern recognition to quickly determine the number of objects in a set, such as seeing 2 things and 2 things and knowing this makes 4 things in all.[†] That is, conceptual subitizing involves "recognizing that a collection of objects is composed of two subcollections and quickly combining their cardinalities to find the cardinality of the collection.[‡]

Solving task K:11 ultimately involves finding the difference 9 - 5 = 4. It is interesting to think about reversing the action of the situation. Suppose the 5 birds that flew away later returned to the tree. Afterwards, there would be 4 + 5 = 9 birds in the tree, the same number as were there at first. We could express this conclusion as

$$(9 - 5) + 5 = 9,$$

where the first operation, 9 **- 5**, shows the action of task K:11, and the second operation, (9 - 5) **+ 5** is the reverse action. This reflects the relationship between addition and subtraction: 9 - 5 is the number that makes 9 when added to 5. More generally, the mathematical relationship between addition and subtraction is that $C - A$ is the unknown addend in $A + ? = C$. Therefore, problems involving subtraction also implicitly involve addition, because subtraction finds an unknown addend. This is why a subtraction calculation is checked by adding.

From an abstract point of view, there's not much more to say than that. Educationally however, there is a lot more to say, because the mathematical relationship between addition and subtraction can play out in real life in conceptually distinct ways. In fact, there are three main meanings or uses for addition and subtraction:

- Adding To/Taking From
- Putting Together/Taking Apart
- Comparing

Furthermore, in a word problem, some quantities in the situation are known while others are initially unknown; the various possibilities for what is known and what is initially unknown combine with the main meanings of addition and subtraction to give a total of fifteen basic situation types for elementary addition and subtraction word problems.

Answer

Now there are 4 birds.

Click here for a student-facing version of the task.

Refer to the Standards

K.OA.A.2; MP.4. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts, Application

Additional notes on the design of the task

- Many students will represent and solve the problem using objects or a drawing. Some students might find the answer by counting on.
- Since kindergarten students should see addition and subtraction equations, student work could be summarized by showing and reading to the student the equation 9 - 5 = 4. (Student writing of equations in kindergarten is also encouraged but not expected in standards.)

In particular, the situation type in task K:11 is called "Take From with Result Unknown."§ Other kindergarten tasks involve the situation types of "Add To with Result Unknown," "Put Together/Take Apart with Total Unknown," and "Put Together/Take Apart with Both Addends Unknown." Kindergarten students represent and solve these problems with objects, fingers, mental images, drawings showing the relationships among the numbers, sounds (for example, claps), acting out situations, verbal explanations, expressions, or equations. These experiences enable students to gain grade-level fluencies and to develop conceptual understandings about addition and subtraction that they will draw upon in grade 1 and beyond.

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: counting out (or drawing) a given number of objects; counting to tell the number of objects; conceptual subitizing; and understanding subtraction as taking from.

Extending the task

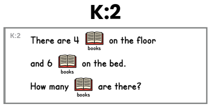
How might students drive the conversation further?

- Students could continue the story of the problem by supposing that a chosen number of the birds decided to fly back to the tree. How many birds would be in the tree then?

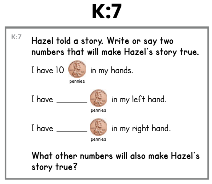
Related Math Milestones tasks

K:10

K:2



K:7



Other tasks in kindergarten that involve the kindergarten situation types are **K:10 Hello, Dogs** ("Add To with Result Unknown"), **K:2 Two Groups of Books** ("Put Together/Take Apart with Total Unknown"), and **K:7 Ten Pennies, Two Hands** ("Put Together/Take Apart with Both Addends Unknown").

In later grades, see the Map of Addition and Subtraction Situations in K–2 Math Milestones.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:11? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:11? In what specific ways do they differ from K:11?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:11 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† See p. 356 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity* (National Research Council. 2009. Washington, DC: The National Academies Press. https://doi.org/10.17226/12519).

‡ Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona, p. 4.

§ For the other situation types, see Table 2, p. 9 of the *Progression* document.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:12 Make Ten and Some More

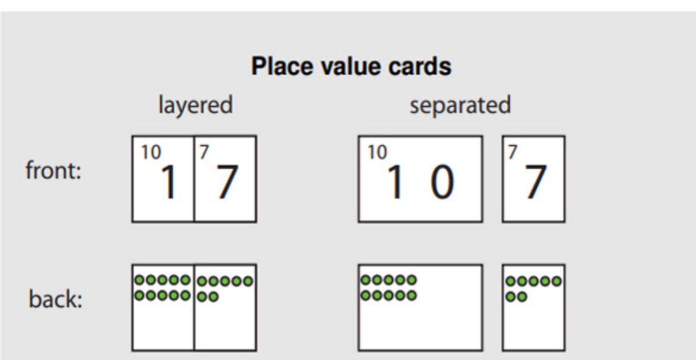
Teacher Notes

Central math concepts

The number names *zero, one, two, three, four, five, six, seven, eight,* and *nine* follow no mathematical system. There's also no system to the corresponding symbol sequence 0, 1, 2, 3, 4, 5, 6, 7, 8, 9—just a few suggestive hints of meaning, such as the symbol 1 consisting of a single stroke, or the symbol 3 having three "points." Then with the larger numbers 10, 11, 12, 13, 14, 15, 16, 17, 18, and 19, systematic patterns appear. These two-digit numbers are a kind of code for naming quantities: for example, 17 refers to a quantity of ten ones and seven more ones. In later grades, students decode a larger number like 463 as the quantity 4 hundreds, 6 tens, and 3 ones. And in later grades still, they decode a decimal number like 3.7 as 3 ones and 7 tenths. The learning progression in understanding and calculating with numbers that are encoded this way is detailed in the relevant *Progression* document.[†] This learning progression begins in kindergarten, where students explore the numbers 11–19 as a foundation for place value in later grades.

Here are some relevant notes about that work excerpted from the *Progression* document (p. 5):

"A difficulty in the English-speaking world is that the words for teen numbers do not make their base-ten meanings evident. For example, 'eleven' and 'twelve' do not sound like 'ten and one' and 'ten and two.' The numbers 'thirteen, fourteen, fifteen, …, nineteen' reverse the order of the ones and tens digits by saying the ones digit first. Also, 'teen' must be interpreted as meaning 'ten' and the prefixes 'thir' and 'fif' do not clearly say 'three' and 'five.' In contrast, the corresponding East Asian number words are 'ten one, ten two, ten three,' and so on, fitting directly with the base-ten structure and drawing attention to the role of ten. Children could learn to say numbers in this East Asian way in addition to learning the standard English number names.



Children can use layered place value cards to see the 10 "hiding" inside any teen number. Such decompositions can be connected to numbers represented with objects and math drawings. When any of the number arrangements is turned over, the one card is hidden under the tens card. Children can see this and that they need to move the ones dots above and on the right side of the tens card.

> K:12 Draw 16 circles. Use a [favorite color] marker for 10 of them. Use a pencil for the rest. [Student draws.]
>
> How many are [favorite color]? How many are in pencil?
>
> Write the missing number: 16 = 10 + ____

Answer

10 circles are [favorite color]. 6 circles are in pencil. 16 = 10 + 3.

Click here for a student-facing version of the task.

Refer to the Standards

K.NBT.A.1; MP.1, MP.2, MP.7. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Concepts

Additional notes on the design of the task

- The task is designed for an interactive format, with the teacher posing the question to a student or group of students.
- Being able to follow the instruction to "Draw 16 circles" is similar to being able to count out 16 objects; task K:12 is designed for use after a student can perform such Counting and Cardinality tasks (CCSS K.CC; *Progression* document[§] pp. 4, 5).
- In the equation 16 = 10 + 6, the operation is on the right-hand side. This decomposition equation emphasizes the meaning of the equal sign ("is the same as").

"The numerals 11, 12, 13, ..., 19 need special attention for children to understand them.... For example, initially, a numeral such as 16 looks like 'one, six,' not '1 ten and 6 ones.' Layered place value cards [see figure] can help children see the 0 'hiding' under the ones place and that the 1 in the tens place really is 10 (ten ones).

"By working with teen numbers in this way in kindergarten, students gain a foundation for viewing 10 ones as a new unit called a ten in Grade 1."

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: saying the count sequence; counting out a number of objects; counting to tell how many; perceptual subitizing; and conceptual understanding of addition.

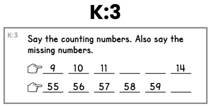
Extending the task

How might students drive the conversation further?

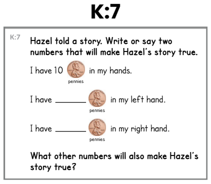
- Students could choose another number besides 16 (such as 13), repeat the drawing step, and say a conclusion statement such as "13 is 10 and 3 more."
- By looking at several cases, students could make a generalization about what the digits of a teen number mean (CCSS MP.8).

Related Math Milestones tasks

K:3

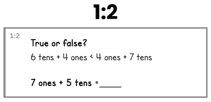


K:7

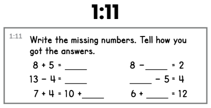


Task **K:3 Say the Numbers (Teens, Decades)** involves reading and saying the count sequence for a range of teen numbers. Task **K:7 Ten Pennies, Two Hands** involves decomposing 10 (whereas task K:12 involves decomposing 16).

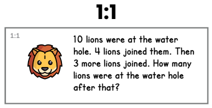
1:2



1:11



1:1



In later grades, task **1:2 Tens and Ones** involves the idea of a tens unit. Task **1:11 Using Properties and Relationships** involves addition and subtraction within 20, in cases like 8 + 5 = ☐ that may involve strategies that rely on the meaning of teen numbers, such as 8 + 5 = 8 + 2 + 3 = 10 + 3 = 13. Task **1:1 Lions at the Watering Hole** involves teen numbers in context.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:12? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:12? In what specific ways do they differ from K:12?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:12 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

† Common Core Standards Writing Team. (2015, March 6). *Progressions for the Common Core State Standards in Mathematics (draft)*. Grades K–5, *Number and Operations in Base Ten*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

§ Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft)*. K, *Counting and Cardinality; K–5, Operations and Algebraic Thinking*. Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

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Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

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Solution Paths

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Language

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- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
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- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:13 Fluency within Five

Teacher Notes

Central math concepts

The numeracy goals for kindergarten can be summarized by listing the relevant CCSS domains and cluster headings for the grade:

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Task K:13 focuses specifically on the fluency goal for kindergarten, which is fluently adding and subtracting within 5 (CCSS K.OA.A.5). This fluency goal is best thought of not as a standalone teaching target, but rather as one expected outcome of a year's worth of extensive, cognitively rich learning experiences with the mathematics articulated by the cluster headings.

Thus, as noted in the relevant *Progression* document[†] (p. 11), "Later in the year, students solve addition and subtraction equations for numbers within 5, for example, 2 + 1 = ☐ or 3 - 1 = ☐, while still connecting these equations to situations verbally or with drawings. Experience with decompositions of numbers and with Add To and Take From situations [see the Teacher Notes for tasks **K:10 Hello, Dogs** and **K:11 Bye-Bye, Birds**] enables students to begin to fluently add and subtract within 5."

Students will generally use Level 1 methods to solve problems like those in task K:13 (see the figure, from p. 6), often using their fingers. Because using fingers is helpful for Level 2 and 3 methods in later grades, "it is important that students in kindergarten develop rapid visual and kinesthetic recognition of numbers to 5 on their fingers. Students may bring from home different ways to show numbers with their fingers and to raise (or lower) them when counting. The three major ways used around the world are starting with the thumb, the little finger, or the pointing finger (ending with the thumb in the latter two cases). Each way has advantages physically or mathematically, so students can use whatever is familiar to them. The teacher can use the range of methods present in the classroom, and these methods can be compared by students to expand their understanding of numbers. Using fingers is not a concern unless it remains at the first level of direct modeling in later grades" (p. 8).

K:13 Write or say the missing numbers.

3 + 1 = ____	2 + 3 = ____
5 + 0 = ____	2 − 2 = ____
4 − 3 = ____	5 − 3 = ____

Answer

Left column: 4, 5, 1. *Right column*: 5, 0, 2.

Click here for a student-facing version of the task.

Refer to the Standards

K.OA.A.5; MP.6. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

Procedural skill and fluency

Additional notes on the design of the task

The task allows for a spoken or written answer, at the student's discretion.

Methods used for solving single-digit addition and subtraction problems

Level 1. Direct Modeling by Counting All or Taking Away. Represent situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Level 2. Counting On. Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. Some method of keeping track (fingers, objects, mentally imaged objects, body motions, other count words) is used to monitor the count.

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as an unknown addend).

Level 3. Convert to an Easier Problem. Decompose an addend and compose a part with another addend.

See Appendix for examples and further details.

The discussion of kindergarten in the *Progression* document ends with this guidance (p. 11): "The kindergarten standards can be stated succinctly, but they represent a great deal of focused and rich interactions in the classroom. This is necessary in order to enable all students to understand all of the numbers and concepts involved. Students who enter kindergarten without knowledge of small numbers or of counting to ten will require extra teaching time in kindergarten to meet the standards. Such time and support are vital for enabling all students to master the Grade 1 standards in Grade 1."

Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: conceptual subitizing; understanding addition as putting together; and understanding subtraction as taking from.

Extending the task

How might students drive the conversation further?

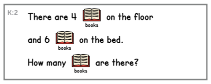
- Students could create situations/word problems corresponding to the sums and differences in the task.
- Focusing on the problem 5 + 0 = ☐, students could look at additional examples (2 + 0, 6 + 0) and state a generalized claim about the result when adding zero (CCSS MP.8).
- Focusing on the problem 2 - 2 = ☐, students could look at additional examples (3 - 3, 1 - 1) and state a generalized claim about the result when subtracting a number from itself (CCSS MP.8).

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:13? Locate 2-3 similar tasks in that unit. How are the tasks you found similar to each other, and to K:13? In what specific ways do they differ from K:13?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:13 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

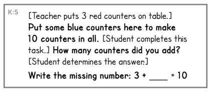
Related Math Milestones tasks

K:2



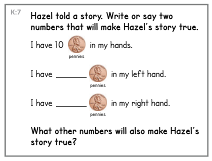
K:2 There are 4 books on the floor and 6 books on the bed. How many books are there?

K:5



K:5 [Teacher puts 3 red counters on table.] Put some blue counters here to make 10 counters in all. [Student completes this task.] How many counters did you add? [Student determines the answer] Write the missing number: 3 + ____ = 10

K:7



K:7 Hazel told a story. Write or say two numbers that will make Hazel's story true.

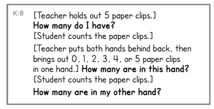
I have 10 pennies in my hands.

I have ______ pennies in my left hand.

I have ______ pennies in my right hand.

What other numbers will also make Hazel's story true?

K:8



K:8 [Teacher holds out 5 paper clips.] How many do I have? [Student counts the paper clips.] [Teacher puts both hands behind back, then brings out 0, 1, 2, 3, 4, or 5 paper clips in one hand.] How many are in this hand? [Student counts the paper clips.] How many are in my other hand?

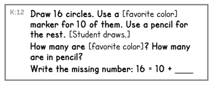
K:10

K:10 5 dogs were playing. Then 3 more dogs came. How many dogs are here now?

K:11

K:11 9 birds were in a tree. 5 birds flew away. How many birds are there now?

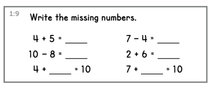
K:12



K:12 Draw 16 circles. Use a [favorite color] marker for 10 of them. Use a pencil for the rest. [Student draws.] How many are [favorite color]? How many are in pencil? Write the missing number: 16 = 10 + ____

Other tasks in kindergarten that call for addition and/or subtraction are **K:2 Two Groups of Books**, **K:5 Adding to Make a Group of Ten**, **K:7 Ten Pennies, Two Hands**, **K:8 Five Behind the Back**, **K:10 Hello, Dogs**, **K:11 Bye-Bye, Birds**, and **K:12 Make Ten and Some More**.

1:9



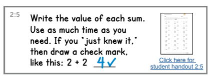
1:9 Write the missing numbers.

4 + 5 = ____ 7 − 4 = ____

10 − 8 = ____ 2 + 6 = ____

4 + ____ = 10 7 + ____ = 10

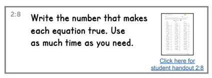
2:5



2:5 Write the value of each sum. Use as much time as you need. If you "just knew it," then draw a check mark, like this: 2 + 2 4✓

Click here for student handout 2:5

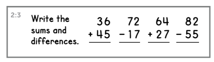
2:8



2:8 Write the number that makes each equation true. Use as much time as you need.

Click here for student handout 2:8

2:3



2:3 Write the sums and differences.

36	72	64	82
+ 45	− 17	+ 27	− 55

In later grades, task **1:9 Fluency within Ten** focuses on the fluency goal for grade 1, and tasks **2:5 Sums of Single-Digit Numbers**, **2:8 Fluency within the Addition Table**, and **2:3 Fluency within 100 (Add/Subtract)** focus on the fluency goals for grade 2.

† Common Core Standards Writing Team. (2011, May 29). *Progressions for the Common Core State Standards in Mathematics (draft): K, Counting and Cardinality; K–5, Operations and Algebraic Thinking.* Tucson, AZ: Institute for Mathematics and Education, University of Arizona.

* Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking.

Anticipating and responding to student thinking about the task

Imagine how students might think about the task, and what you might see and hear while they work.

On this page, you can write your thoughts on the following questions.

Solution Paths

- What solution paths might you expect to see?
- What representations might you see? What correspondences between those representations might be noticed by students (or be worth pointing out to students) and discussed by them?
- What misconceptions or partial understandings might be revealed as students work on the task? How could you respond to these positively and productively?

Language

- What might you expect to hear from students engaged with the task? What does that language reveal about their mathematical thinking, and how might you respond to different ways of thinking?
- If students are using early English or using multiple languages in an integrated communication system, how might you help their classmates see those mathematical ideas as valuable?
- Even when using nascent language, students are thinking and communicating their thinking. What might it look like to respond positively and productively to the mathematics in their thinking before giving feedback on the language used?

Identity, Agency, and Belonging

- How can you engage students' interests, experiences, or funds of knowledge?
- How can you build students' self-confidence as learners, thinkers, and doers of mathematics?
- What choices are there for a student to make in the task? How can you build students' agency to the point where they notice and make these choices to solve problems?
- How might one use feedback to build student agency? Where might there be opportunities to build students' self-confidence?

K:14 Animals from Land and Sea

Teacher Notes

Central math concepts

Counting always involves a choice about what objects we will "count as the same." For example, if we say that there are 9 *animals* in this task, then we are ignoring all differences between the animals: where they live, how big they are, how many legs they have, and so on. Often these choices are instinctive; for example, when a kindergarten student counts a collection of pennies on a table, the student may not even reflect that they are ignoring the difference between pennies that are heads-side-up and pennies that are tails-side-up. What counts—the category for counting; the unit—is simply "pennies." In task **K:14 Animals from Land and Sea**, thinking about the categories for counting is central to the task. As such, this task signals the beginning of a student's study of categorical data, and more importantly the task foreshadows later grades, when choosing a unit will become a prominent element of success in arithmetic reasoning and problem solving.

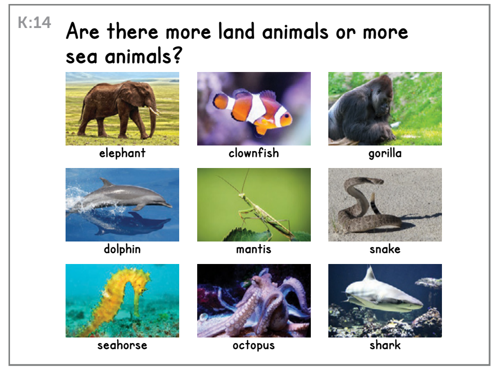
Relevant prior knowledge

The following mathematics knowledge may be activated, extended, and deepened while students work on the task: counting to tell the number of objects; and identifying whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, for example by using matching and counting strategies.

Extending the task

How might students drive the conversation further?

- Students might ask (or you might ask the students), "What else could we count?" Students might wonder, "How many animals in all? How many eyes are there?" Students could then use counting to find the answers.
- Students could relate categories and units: for example, they could say, "4 land animals + 5 sea animals = 9 animals."



Answer

There are more sea animals than land animals.

Click here for a student-facing version of the task.

This task is designed so that students could work with cutouts of the animal images as they classify and count.

Refer to the Standards

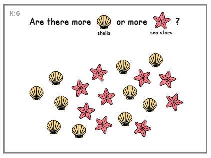
K.MD.B.3; MP.2. Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.

Aspect(s) of rigor:

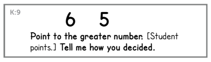
Application

Related Math Milestones tasks

K:6

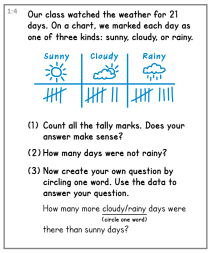


K:9

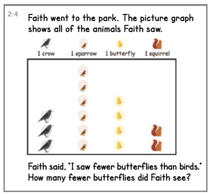


Other tasks for kindergarten that involve comparisons are **K:6 More Shells or More Stars?** and **K:9 Compare 6 and 5**.

1:4



2:4



Task **1:4 Analyzing Weather Data** continues the story of categorical data into grade 1, and task **2:4 Animals in the Park** develops the story further in grade 2.

Additional notes on the design of the task

- The task does not require students to read the animal name words.
- The task does not require students to know facts about the animals ahead of time. Note that the images provide some clues about the animals' habitats. For other cases, students could be informed of the animal's habitat, or students' varied funds of knowledge could be leveraged. The task could also be integrated with science units (see NGSS, Interdependent Relationships in Ecosystems: Animals, Plants, and Their Environment).
- In whatever way students classify the animals, what is important mathematically is for students to proceed from that classification to compare the number of animals in each category.

Curriculum connection

1. In which unit of your curriculum would you expect to find tasks like K:14? Locate 2-3 similar tasks in that unit. How are the tasks similar to each other, and to K:14? In what specific ways do they differ?
2. Thinking about the curriculum unit you identified, at what point in the unit might a task like K:14 help students converge toward grade-level thinking about the important mathematics in the task? What factors would you consider in choosing when to use such a task in the unit?*

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