## Math Milestones<sup>™</sup> — Kindergarten



How many blocks? [Student tells how many.]



[Teacher slowly rearranges.] If you count the blocks, how many do you think there will be?



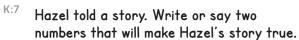
There are 4 on the floor

and 6 on the bed.

How many are there?

Say the counting numbers. Also say the

(<del>\*\*</del> 9 10 11 14



Are there more or more



I have \_\_\_\_\_ in my left hand.

I have \_\_\_\_\_ in my right hand.

What other numbers will also make Hazel's story true?

K:12 Draw 16 circles. Use a [favorite color]

How many are [favorite color]? How many are in pencil?

marker for 10 of them. Use a pencil for

were playing.

How many ore here now?

How many or are there now?

the rest. [Student draws.]

K:13 Write or say the missing numbers.

3 + 1 = \_\_\_\_

Then 3 more came.

9 🧼 were in a tree.

5 ends flew away.

Write the missing number: 16 = 10 + \_\_\_\_

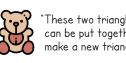
Are both of the bears correct? [Student uses manipulatives to answer.]



missing numbers.







These two triangles can be put together to make a new triangle.

[Teacher puts 3 red counters on table.] Put some blue counters here to make task.] How many counters did vou add? [Student determines the answer.]

10 counters in all. [Student completes this Write the missing number: 3 + \_\_\_\_ = 10 [Teacher holds out 5 paper clips.] How many do I have?

[Student counts the paper clips.]

[Teacher puts both hands behind back, then brings out 0, 1, 2, 3, 4, or 5 paper clips in one hand.] How many are in this hand? [Student counts the paper clips.]

How many are in my other hand?

K:9

Point to the greater number. [Student points.] Tell me how you decided.

Are there more land animals or more sea animals?







2 + 3 = \_\_\_\_









## Math Milestones<sup>™</sup> Task List — Kindergarten

# The 14 Math Milestones™ tasks for kindergarten have been carefully crafted to embody kindergarten mathematics on one page.

| K:1  | How Many Blocks?                 | $\widehat{\mathbb{A}}$       | СР | K.CC.B.4            |
|------|----------------------------------|------------------------------|----|---------------------|
| K:2  | Two Groups of Books              |                              | СА | K.OA.A.2            |
| K:3  | Say the Numbers (Teens, Decades) |                              | Р  | K.CC.A.1, 2         |
| K:4  | Bears Talk About Shapes          | $\widehat{\mathcal{L}}_{U}$  | С  | K.G.A.2, K.G.B.4,6  |
| K:5  | Adding to Make a Group of Ten    | $\widehat{\mathcal{L}}_{U}$  | С  | K.OA.A.4            |
| K:6  | More Shells or More Stars?       |                              | СР | K.CC.B.5            |
| K:7  | Ten Pennies, Two Hands           | $\widehat{\mathcal{L}}_{U}$  | СР | K.OA.A.3, 4         |
| K:8  | Five Behind the Back             | $\widehat{\mathcal{L}}_{U}$  | С  | K.OA.A              |
| K:9  | Compare 6 and 5                  |                              | СР | K.CC.B.4c, K.CC.C.7 |
| K:10 | Hello, Dogs                      |                              | СА | K.OA.A.2            |
| K:11 | Bye-Bye, Birds                   |                              | СА | K.OA.A.2            |
| K:12 | Make Ten and Some More           |                              | С  | K.NBT.A.1           |
| K:13 | Fluency within Five              |                              | Р  | K.OA.A.5            |
| K:14 | Animals from Land and Sea        | $\widehat{\mathbb{Q}}_{\mu}$ | Α  | K.MD.B.3            |

C = Task has a conceptual focus. P = Task has a procedural skill & fluency focus. A = Task has an application focus. Task is designed for use with manipulatives or objects. Students might also use manipulatives to support their work on other tasks.

#### Standards for Mathematical Practice

| MP.1 Make sense of problems and persevere in solving them.            | K:5-8, K:12                |
|---|----------------------------|
| MP.2 Reason abstractly and quantitatively.                            | K:1, K:5, K:8, K:9, K:12   |
| MP.3 Construct viable arguments and critique the reasoning of others. | K:9                        |
| MP.4 Model with mathematics.  | K:2, K:7, K:10, K:11, K:14 |
| MP.5 Use appropriate tools strategically.                             | K:4, K:5                   |
| MP.6 Attend to precision.   | K:3, K:6, K:13             |
| MP.7 Look for and make use of structure.                              | K:5, K:12                  |
| MP.8 Express regularity in repeated reasoning.                        | K:3, K:7                   |

Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.



Math Milestones<sup>™</sup> was created by Jason Zimba, John W. Staley, Elizabeth Meier, Sandra Alberti, Harold Asturias, and Phil Daro.

Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking. Student work on tasks could be collected in student portfolios.

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1:1



10 lions were at the water hole. 4 lions joined them. Then 3 more lions joined. How many lions were at the water hole after that?

| 1:5 | Tyler has 6 more grapes than Zoey. |
|-----|------------------------------------|
|     | Zoey has 8 grapes. How many grapes |
|     | does Tyler have?                   |

| Equation model:   |         |
|-------------------|---------|
| Answer: Tyler has | grapes. |

Write the missing numbers. Tell how you got the answers.

1:2 True or false?

6 tens + 4 ones < 4 ones + 7 tens

7 ones + 5 tens =



1:6

I have 24 straws in a jar. I have 30 straws in a bag. How many straws do I have? Grace tried to blow out 15 candles on her birthday cake. Grace blew out 9 candles. How many candles are still lit?

| Equation models | :       |     |       |      |
|-----------------|---------|-----|-------|------|
| Answer:         | candles | are | still | lit. |

Using a paper clip as a unit of length, draw a straight line 7 units long.



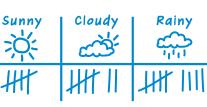
If the class works hard, our teacher will put a marble in a jar. We will have a party when there are 10 marbles in the jar. Today there are 6 marbles in the jar. How many marbles do we need for a party?

1:13



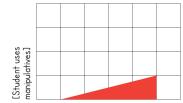
When I fell asleep last night, there were 8 icicles outside my window. When I woke up this morning, there were 3 icicles. How many icicles fell while I slept?

Our class watched the weather for 21 days. On a chart, we marked each day as one of three kinds: sunny, cloudy, or rainy.



90 - 40 = 9 tens - 4 tens =

One statement below is false. Find the false statement. How did you decide?



A square can be created using triangles like this one.

- (1) Count all the tally marks. Does your answer make sense?
- (2) How many days were not rainy?
- (3) Now create your own question by circling one word. Use the data to answer your question.

How many more cloudy/rainy days were (circle one word)

there than sunny days?

Write the missing numbers.

1:10

None of these are squares.





The shaded part of the circle is one fourth of the whole circle.



# The 14 Math Milestones™ tasks for grade 1 have been carefully crafted to embody grade 1 mathematics on one page.

| 1:1  | Lions at the Watering Hole         |                              | СА | 1.OA.A.2, 1.OA, |
|------|------------------------------------|------------------------------|----|-----------------|
| 1:2  | Tens and Ones                      |                              | С  | 1.NBT.B         |
| 1:3  | Paper Clip Length Units            |                              | СА | 1.MD.A          |
| 1:4  | Analyzing Weather Data             |                              | Α  | 1.MD.C.4        |
| 1:5  | Tyler's Grapes                     |                              | CA | 1.OA.A.1, 1.OA  |
| 1:6  | Two Groups of Straws               |                              | РΑ | 1.NBT.C, 1.OA.A |
| 1:7  | Class Marble Jar                   |                              | CA | 1.OA.A.1, 1.OA  |
| 1:8  | Subtracting Units                  |                              | С  | 1.NBT.C.6       |
| 1:9  | Fluency within Ten                 |                              | Р  | 1.OA.C.6        |
| 1:10 | Two-Digit Addition                 |                              | СР | 1.NBT.C.4       |
| 1:11 | Using Properties and Relationships |                              | СР | 1.OA.B          |
| 1:12 | Blowing Out Candles                |                              | CA | 1.OA.A.1, 1.OA  |
| 1:13 | Falling Icicles                    |                              | CA | 1.OA.A.1, 1.OA  |
| 1:14 | Shape True/False                   | $\widehat{\mathbb{A}}_{\mu}$ | С  | 1.G.A           |

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#### **Standards for Mathematical Practice**

| MP.1 Make sense of problems and persevere in solving them.            | 1:2, 1:4-7, 1:11-14        |
|---|----------------------------|
| MP.2 Reason abstractly and quantitatively.                            | 1:1, 1:3-5, 1:12           |
| MP.3 Construct viable arguments and critique the reasoning of others. | 1:11, 1:14                 |
| MP.4 Model with mathematics.  | 1:1, 1:4-7, 1:12, 1:13     |
| MP.5 Use appropriate tools strategically.                             | 1:3, 1:14                  |
| MP.6 Attend to precision.   | 1:2, 1:9-11                |
| MP.7 Look for and make use of structure.                              | 1:2, 1:8, 1:10, 1:11, 1:14 |
| MP.8 Express regularity in repeated reasoning.                        | 1:8                        |

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Avi made a paper chain. Then Avi added 29 more links to the paper chain. Now there are 52 links in the paper chain. How many links were in the paper chain before?



Use as much time as you need. If you 'just knew it,' then draw a check mark, like this: 2 + 2 4



A grass snake is 28 inches long. A rat snake is 74 inches long. How much longer is the rat snake?

Draw a diagram to illustrate your solution. Label the diagram with numbers.

2:2 (1) True or false?

(a) 2 hundreds + 3 ones > 5 tens + 9 ones

**(b)** 9 tens + 2 hundreds + 4 ones < 924

(c) 456 < 5 hundreds

(2) Write the number that makes each statement true.

(a) 7 ones + 5 hundreds = \_\_\_\_

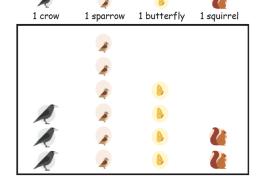
**(b)** 14 tens = \_\_\_\_

(c) 90 + 300 + 4 = \_\_\_\_

Write the sums and differences.

36 72 64 82 + 45 - 17 + 27 - 55

Faith went to the park. The picture graph shows all of the animals Faith saw.



Faith said, 'I saw fewer butterflies than birds.' How many fewer butterflies did Faith see?

A rope is 32 feet long. The rope is cut into two pieces. One piece is 3 feet long. How long is the other piece?

Equation model: \_\_\_\_\_\_ feet

(1) Write the number that makes the statement true.

6 hundreds + 3 tens + 4 ones

= 5 hundreds + \_\_\_\_\_ tens + 4 ones.

(2) How do you know your statement is true?

(3) Look for connections between your statement and this subtraction problem. What connections can you see?

TO THE PROPERTY OF THE PROPERT

634

152

-482

Write the number that makes each equation true. Use as much time as you need.

A farmer said, "Last night some deer came and ate 16 of my cabbages. Now I only have 38 cabbages." How many cabbages were there before the deer came?

Equation model: \_\_\_\_\_ cabbages.

2:10 Check the subtraction by adding. 946 - 678 = 268 At recess there was a jump-rope contest.



I won because I jumped 25 more times than Catherine.

How many times did Catherine jump?

I jumped 81 times.

Equation model: \_\_\_\_\_\_ times.

2:13 Marlon and Malia went apple-picking.



You picked 13 fewer apples than I did.



How many apples did Malia pick?

Equation model: \_\_\_\_\_ apples.

2:14 Zariah got one answer wrong.

- (1) Which answer did Zariah get wrong?
- (2) Correct Zariah's wrong answer.
- (a) Show how the rectangle can be divided into 15 squares.



(b) 2 halves make one whole.

(c) Draw a triangle. All three sides of your triangle must have different lengths.



# The 14 Math Milestones™ tasks for grade 2 have been carefully crafted to embody grade 2 mathematics on one page.

| 2:1  | Paper Chain                       | CAP | 2.OA.A.1, 2.NBT.B.5 |
|------|-----------------------------------|-----|---------------------|
| 2:2  | Place Value to Hundreds           | С   | 2.NBT.A             |
| 2:3  | Fluency within 100 (Add/Subtract) | Р   | 2.NBT.B.5           |
| 2:4  | Animals in the Park               | Α   | 2.MD.D.10           |
| 2:5  | Sums of Single-Digit Numbers      | Р   | 2.OA.B.2            |
| 2:6  | Cutting a Rope                    | CA  | 2.MD.B.5, 2.MD.B    |
| 2:7  | Subtraction Regrouping            | СР  | 2.NBT.B.7, 2.NBT.B  |
| 2:8  | Fluency within the Addition Table | Р   | 2.OA.B.2            |
| 2:9  | Disappearing Cabbages             | CAP | 2.OA.A.1, 2.NBT.B.5 |
| 2:10 | Three-Digit Addition/Subtraction  | СР  | 2.NBT.B.7           |
| 2:11 | Grass Snake vs. Rat Snake         | CAP | 2.MD.B, 2.NBT.B.5   |
| 2:12 | Jump-Rope Contest                 | CAP | 2.OA.A.1, 2.NBT.B.5 |
| 2:13 | Apple-Picking                     | CA  | 2.OA.A.1            |
| 2:14 | Correcting a Shape Answer         | С   | 2.G.A               |



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A = Task has an application focus.

#### **Standards for Mathematical Practice**

MD 1 Make some of problems and personers in solving them

| MP.I Make sense of problems and persevere in solving them.           | 2:1, 2:2, 2:5-9, 2:11-14    |
|--|-----------------------------|
| MP.2 Reason abstractly and quantitatively.                           | 2:6, 2:7, 2:11-13           |
| MP.3 Construct viable arguments and critique the reasoning of others | s. 2:7, 2:14                |
| MP.4 Model with mathematics.   | 2:1, 2:4, 2:6, 2:9, 2:11-13 |
| MP.5 Use appropriate tools strategically.                            | 2:14                        |
| MP.6 Attend to precision.  | 2:2-5, 2:7, 2:8, 2:10       |
| MP.7 Look for and make use of structure.                             | 2:2, 2:3, 2:7, 2:10, 2:14   |
| MP.8 Express regularity in repeated reasoning.                       | 2:2                         |

2-1 2-2 2-5 0 2-11 14

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There are five teams in the volleyball league. Every team has six players. How many players are in the volleyball league?

Equation model: \_\_\_\_\_

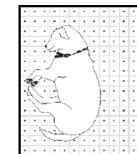
Answer:

Jasmine bought 45 corn seeds. She arranged the seeds into piles of 9 seeds each. How many piles were there?

Equation model: \_\_\_\_\_



3:2



The picture shows a dog sleeping on a rug. The rug design is a rectangular array of squares with a dot in each square.

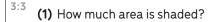
Below are four expressions. One expression equals the total number of dots in the rug design. Which expression equals the total number of dots in the rug design? Tell how you decided.

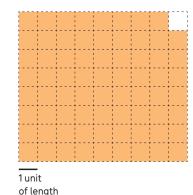
12 × 14, 11 × 14, 12 × 15, 11 × 15

Our class picked up litter on the playground. One student wrote tally marks to record the things we picked up.

Show the data another way by drawing a scaled picture graph in which 1 picture stands for 10 things picked up.

Using what you know about fractions, decide which is greater,  $\frac{1}{73}$  or  $\frac{1}{41}$ . Tell how you decided.





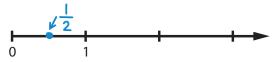
(2) Using a ruler, draw a rectangle with area 28 square centimeters. Write the length and width of your rectangle.

Length: \_\_\_\_\_ Width: \_\_\_\_

3:7 Here is a list of numbers. Where does each number belong on the number line?

$$\frac{1}{2}$$
,  $\frac{1}{4}$ , 2,  $\frac{5}{4}$ ,  $\frac{2}{2}$ ,  $\frac{3}{2}$ ,  $\frac{6}{4}$ ,  $\frac{2}{4}$ ,  $\frac{3}{1}$ 

Draw a dot to show the location of each number. Label each dot. The first number in the list has been located for you.



- (1) Name two attributes that are shared by triangles and squares.
  - (2) Name a category of shapes that includes triangles and squares and also includes other shapes that have both of the attributes you named.

- Our class painted pictures. The teacher will hang the pictures on 4 bulletin boards. The teacher will hang the same number of pictures on each board. How many pictures will be on each board? There are 32 pictures to hang.
- Alice forgot what  $7 \times 8$  equals. Alice knows that  $5 \times 8 = 40$  and  $2 \times 8 = 16$ .
  - (1) Write a sentence to tell Alice how she can find the value of  $7 \times 8$  by using the two facts she knows.
  - (2) Draw a diagram that could help Alice understand why your method works.
  - (3) Choose two numbers other than 7 and 8, and try using your method to multiply them. Will your method work for any pair of factors? Say why you think so.
- Steven, Hawa, and 4 more friends went to the park. Steven brought 24 water balloons. Hawa brought 24 water balloons. Nobody else brought water balloons. The 6 friends shared all the water balloons equally. How many water balloons did each friend get?
- 3:12 Write the value of each product. Use as much time as you need. If you "just knew it," then draw a check mark, like this:

2×2 4√



Click here for student handout 3:12

3:13 Write the number that makes each equation true. Use as much time as you need.



Click here for student handout 3:13

3:14 Write the sums and differences.

With pencil and paper

351 264 625 831 + 472 + 438 - 261 - 444 Mentally 800 - 300

240 + 540 365 – 165

612 – 13

## The 14 Math Milestones™ tasks for grade 3 have been carefully crafted to embody grade 3 mathematics on one page.

3:1 Volleyball Players C A 3.OA.A.3, 3.OA.A

3:2Hidden Rug DesignC3.OA.A.13:3Length and Area QuantitiesC3.MD.C

**3:4** Corn Seeds C A 3.OA.A.3, 3.OA.A

3:5Playground CleanupA3.MD.B.33:6Unit Fraction IdeasC3.NF.A3:7Locating Numbers on a Number LineC3.NF.A

3:9 Bulletin Board Pictures C A 3.OA.A.3, 3.OA.A
3:10 Alice's Multiplication Fact C P 3.OA.B.5, 3.MD.C.7b

3:11Water BalloonsA3.OA.D.83:12Products of Single-Digit NumbersP3.OA.C.7

**3:13** Fluency within the Multiplication Table P 3.OA.C.7, 3.OA.A.4, 3.OA.A, B, C

3.G.A.1

3:14 Fluency within 1000 (Add/Subtract) P 3.NBT.A.2

C = Task has a conceptual focus.

P = Task has a procedural skill & fluency focus.

Shape Attributes and Categories

A = Task has an application focus.

#### **Standards for Mathematical Practice**

MP.1 Make sense of problems and persevere in solving them. 3:1, 3:4, 3:6-12 **MP.2** Reason abstractly and quantitatively. 3:1-4, 3:5, 3:7, 3:9 MP.3 Construct viable arguments and critique the reasoning of others. 3:2, 3:6, 3:10 MP.4 Model with mathematics. 3:1, 3:4, 3:5, 3:9, 3:11 **MP.5** Use appropriate tools strategically. 3:3, 3:7 MP.6 Attend to precision. 3:2, 3:3, 3:5, 3:7, 3:12-14 MP.7 Look for and make use of structure. 3:2, 3:3, 3:6-8, 3:10, 3:13, 3:14 **MP.8** Express regularity in repeated reasoning. 3:6, 3:10

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4:1 A tablespoon holds 15 ml of olive oil, which is 3 times as much as a teaspoon holds. How many ml of olive oil does a teaspoon hold?

Equation model: \_\_\_\_\_

Answer: \_\_\_\_\_



- (1) Find the three missing lengths and write them on the diagram. Compare answers with a classmate.

  137
  7 | 959
  -700
  259
- (2) What is the total area of the diagram? \_\_210
- (3) Look for connections between the diagram and the division problem.

  What connections can you see?
- Everyone in class measured the length of their pencil. Here are the measurements:



- (1) How many pencils were measured?
- (2) How much longer was the longest pencil than the shortest pencil?
- (3) Could two of the pencils be laid end to end to make a total length of 1 foot?
- 4:4 (1) Compare  $\frac{5}{9}$  to  $\frac{4}{7}$ . First do it by making equal denominators. Then do it by making equal numerators.
  - (2) Ariana said, " $\frac{300}{400}$  looks greater than  $\frac{3}{4}$ . How can they be the same size?" Write or say an explanation that could help Ariana understand why  $\frac{300}{400}$  and  $\frac{3}{4}$  are the same size.
  - (3) Which is closer to 1 on a number line,  $\frac{4}{5}$  or  $\frac{5}{4}$ ? Tell how you decided. Draw a number line and show  $\frac{4}{5}$  and  $\frac{5}{4}$  accurately on the number line.

- 4:5 (1a-f) Write the values of the products. Compare answers with a classmate.
  - (1g) Which answer is twice as much as the answer for (e)?
  - (1h) Which answer is six times as much as the answer for (a)?
  - (1i) Which two answers are equal?
  - (2) Zoe was reading her math book.  $9 \times \frac{2}{9}$ She saw the equation  $6 \times (4 + \frac{1}{2}) = 24 + 3$ . She said, "I don't get it—where did the 24 and the 3 come from?" Write an explanation that could answer Zoe's question.
- Grandpa took a jar of pennies to the bank. He said, "I'd like nickels for this, please." The bank teller poured the pennies into a counting machine. "Eighty-seven dollars and forty-two cents," said the teller. (1) How many nickels did Grandpa get? (2) Check your answer with an estimate.



- ions Dood ooob
- 4:7 Write the values of the expressions. Read each completed equation aloud.

$$\frac{1}{10} + \frac{3}{100} =$$
 (fraction) = (decimal)

$$\frac{1}{8} + \frac{5}{8} - \frac{3}{8} =$$

- 4:8 L is a line, R is a ray, and T is a triangle. True or false:
  - (1) Line L is a line of symmetry for triangle T.



- (2) Line L intersects ray R.
- (3) Triangle T has two angles measuring less than 90 degrees.

4:9 In gym it was fitness day. Students ran laps around the gym.



I ran  $1\frac{2}{3}$  more laps than Catherine.

$$\int I \operatorname{ran} 6 \frac{1}{3} \operatorname{laps}.$$

How many laps did Catherine run?

4:10 Write the values of the products and quotients. Check the quotients by multiplying.

| Mentally | 40 × 20                                 |         | Wit             | With pencil and paper |         |  |
|----------|---|---------|-----------------|-----------------------|---------|--|
|          | 30 × 11<br>12 × 60<br>5 × 19<br>480 ÷ 8 | 6,<br>× | 132<br><u>6</u> | 48<br>× 39            | 7 8,722 |  |

- 4:11 A cook in the school kitchen uses 6 oz of cheese to make a pizza. The kitchen has 45 lb of cheese. How many pizzas will that make?
- 4:12 The pickup truck can carry  $1\frac{3}{5}$  tons. The super hauler truck can carry 300 times as much. How many tons can the super hauler truck carry?



Pickup Truck

- 4:13 (1) A red rectangle has length L = 12 in and width W = 6 in. Use the formula  $A = L \times W$  to find the area of the red rectangle.
  - (2) A blue rectangle has length 1 ft and width  $\frac{1}{2}$  ft. Draw a picture to show that two copies of the blue rectangle make one square foot. Based on your picture, what is the area of the blue rectangle?
  - (3) Do the red rectangle and the blue rectangle have equal areas? Tell how you decided.
- <sup>4:14</sup> 540,909 + 87,808 5,864 + 2,556 = ?

## The 14 Math Milestones™ tasks for grade 4 have been carefully crafted to embody grade 4 mathematics on one page.

| 4:1  | A Tablespoon of Oil                           | СА  | 4.OA.A.2                 |
|------|---|-----|--------------------------|
| 4:2  | Multi-Digit Division Concepts                 | С   | 4.NBT.B.6                |
| 4:3  | Pencil Data                                   | Α   | 4.MD.B.4                 |
| 4:4  | Comparing Fractions with Equivalence          | С   | 4.NF.A                   |
| 4:5  | Fraction Products and Properties              | С   | 4.NF.B.4a, 4b            |
| 4:6  | Jar of Pennies                                | ΑP  | 4.OA.A.3                 |
| 4:7  | Fraction Sums and Differences                 | СР  | 4.NF.B.3a-c, 4.NF.C.5, 6 |
| 4:8  | Shapes with Given Positions                   | С   | 4.MD.C, 4.G.A            |
| 4:9  | Fitness Day                                   | CA  | 4.NF.B.3d                |
| 4:10 | Calculating Products and Quotients            | Р   | 4.NBT.B                  |
| 4:11 | School Kitchen                                | Α   | 4.MD.A.2, 4.NBT.B.5      |
| 4:12 | Super Hauler Truck                            | C A | 4.NF.B.4c, 4.OA.A.2      |
| 4:13 | Area Units                                    | С   | 4.MD.A.3                 |
| 4:14 | Fluency with Multi-Digit Sums and Differences | Р   | 4.NBT.B.4                |

C = Task has a conceptual focus.

P = Task has a procedural skill & fluency focus.

A = Task has an application focus.

#### **Standards for Mathematical Practice**

| MP.1 Make sense of problems and persevere in solving them.            | 4:6, 4:11                      |
|---|--------------------------------|
| MP.2 Reason abstractly and quantitatively.                            | 4:1, 4:2, 4:3, 4:6             |
| MP.3 Construct viable arguments and critique the reasoning of others. | 4:4, 4:5, 4:13                 |
| MP.4 Model with mathematics.  | 4:1, 4:3, 4:6, 4:9, 4:11, 4:12 |
| MP.5 Use appropriate tools strategically.                             | 4:5(2), 4:8                    |
| MP.6 Attend to precision.   | 4:4, 4:8, 4:10, 4:13, 4:14     |
| MP.7 Look for and make use of structure.                              | 4:2, 4:5, 4:7, 4:8, 4:10, 4:14 |
| MP.8 Express regularity in repeated reasoning.                        | 4:4, 4:5, 4:7                  |

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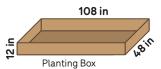


- A school needed 240 four-packs of juice boxes for a field trip. However, the school accidentally bought 240 six-packs of juice boxes. How many extra juice boxes did the school buy?
- After a hurricane, the 12 residents of a nursing home didn't have any clean water to drink. Their neighbors donated 40 gallons of bottled water, which would provide \_\_\_\_ gallons for each resident.



A neighborhood garden will have 6 wooden planting boxes. Every box will have the same shape (see diagram). Soil can be bought by the truckload; a truckload is 54 ft<sup>3</sup> of soil. How many truckloads of soil will fill all of the boxes?





- 5:4 (1) Circle T for true or F for false.
  - (a) 9 thousandths + 5 hundredths > 3 hundredths + 2 tenths T F (b) 92 hundredths + 4 thousandths

T F

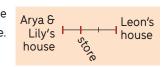
> 0.924 **(c)** 0.456 < 0.5

- T F
- (2) Write each number in the requested form.
  - (a) 7 thousandths + 5 tenths = \_\_\_\_ (decimal)
  - **(b)** 0.1 tenths = \_\_\_\_ (decimal)
  - (c)  $\frac{2}{100} + \frac{5}{1000} =$  (decimal) = \_\_\_\_ (fraction in lowest terms)
- 5:5 Write the requested values.

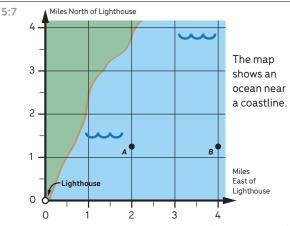
| $4087 \times 53 = ?$<br>$246 \times 914 = ?$<br>$9744 \div 12 = ?$<br>$1461 \div 6 = ?$<br>4 - (8 - 4) = ? | $\frac{1}{10} \div 10 = ?$ $\frac{7}{8} \times \frac{5}{3} = ?$ $8 \times ? = 73$ $3 \div \frac{1}{8} = ?$ $\frac{1}{2} + \frac{1}{3} - \frac{1}{5} = ?$ $\frac{1}{3} \div (6 \times 5) = ?$ | $0.4 \times 0.9 = ?$<br>$0.75 \div 0.01 = ?$<br>$0.63 \div 0.3 = ?$<br>0.86 + 0.4 = ?<br>0.72 - 0.17 = ?<br>0.02 + 0.2 = ?<br>0.8 - 0.55 = ?<br>637 - 1.31 = ? |
|--|--|--|
|  |  |  |

5:6 (1) Arya and Lily's house is  $\frac{1}{5}$  mile from the store. (a) Arya ran  $\frac{1}{3}$  of the

way from her house to



the store. How far, in miles, did Arya run? **(b)** Lily ran  $\frac{2}{3}$  of the way from her house to the store. How far, in miles, did Lily run? **(2)** It is  $\frac{2}{5}$  mile from Leon's house to the store. **(a)** Leon ran  $\frac{1}{3}$  of the way from his house to the store. How far, in miles, did Leon run? **(b)** Compare how far Leon and Lily ran; what do you notice, and why is it true?

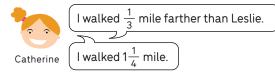


Shipwrecks are at locations  $A(2, 1\frac{1}{4})$  and  $B(4, 1\frac{1}{4})$ . Shipwrecks are also at locations  $C(4, 3\frac{1}{2})$  and  $D(2, 3\frac{1}{2})$ . (1) Mark C and D on the map and shade rectangle ABCD. (2) Some believe there is sunken treasure in the region you shaded. How large is that region in mi<sup>2</sup>?

A scalene triangle is a triangle in which the sides all have different lengths. Thinking about this, Alana decided there should also be a name for quadrilaterals in which the sides all have different lengths. She said, "I'll name them after myself." She defined an αlαnα-gon to be a quadrilateral in which the four sides all have different lengths.

(1) Draw an example of an alana-gon. (2) True or false: (a) All squares are alana-gons. (b) No trapezoids are alana-gons.

5:9 On Saturday there was a walkathon.



How many miles did Leslie walk?

- 5:10 (1) Solve:  $\frac{1}{3} = 0.1 + ?$ 
  - (2) Is there a number greater than  $\frac{1}{5}$  and less than  $\frac{1}{4}$ ? If you think so, find such a number. If you think there is no such number, explain why.
  - (3) Show one of the above problems and its solution on a number line.
- 5:11 Juliet said, "I'm thinking of a rectangle. Its area is 1 square unit. Its perimeter is more than 1 million units."
  - (1) Is Juliet thinking of something possible or impossible? Use math to decide for sure.
  - (2) Explain your reasoning to your classmates. Revise your explanation based on suggestions from your classmates.
- 5:12 Before it rained, the teacher went outside and placed identical baking pans on the ground. After it rained, the teacher brought the pans inside, and students measured how much water was collected in each pan.

  Water Collected



If all the water collected were shared equally among the pans, how much water would be in each pan?

- 5:13 In a snack shop there is a frozen yogurt machine. When there is 3 l of frozen yogurt in the machine, the machine is  $\frac{1}{3}$  full. How much frozen yogurt is in the machine when it is  $\frac{1}{4}$  full?
- 5:14 Brandon was reading his math book. He saw the equation  $\frac{3}{4} \times (4 + \frac{1}{2}) = 3 + \frac{3}{8}$ . He said, "I don't get it—where did the 3 and the  $\frac{3}{8}$  come from?" Write an explanation that could answer Brandon's question.

### The 14 Math Milestones™ tasks for grade 5 have been carefully crafted to embody grade 5 mathematics on one page.

Juice Box Mixup Water Relief

Neighborhood Garden

Place Value to Thousandths

Calculating

Corner Store

Shipwrecks

Alana's New Shape Category

**5:9** Walkathon

5:10 Number System, Number Line

5:11 Juliet's Rectangle

**5:12** Rain Measurements

5:13 Frozen Yogurt Machine

5:14 Brandon's Equation

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P = Task has a procedural skill & fluency focus.

A = Task has an application focus.

C A 5.OA.A.2, 5.NBT.B.5

C A 5.NF.B.3

5.MD.A, B

5.NBT.A

5.NBT.B, 5.NF.A, B

C A 5.NF.B.4a, 5.NF.B.6

C A 5.NF.B.4b, 5.G.A

5.G.B

A P 5.NF.A.1, 2

5.NF.A.1

5.NF.B

5.MD.B

5.NF.B.6, 7

5.NF.B.4a



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**Standards for Mathematical Practice** 

MP.1 Make sense of problems and persevere in solving them.

**MP.2** Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

**MP.5** Use appropriate tools strategically.

MP.6 Attend to precision.

MP.7 Look for and make use of structure.

**MP.8** Express regularity in repeated reasoning.

5:3, 5:6, 5:10, 5:11, 5:13

5:2, 5:6, 5:7, 5:9, 5:11-13

5:6, 5:8, 5:10, 5:11, 5:14

5:1-3, 5:6, 5:7, 5:9, 5:12, 5:13

5:10, 5:13, 5:14

5:4, 5:5, 5:8, 5:11

5:1, 5:4-7, 5:14

5:6, 5:8, 5:11

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- 6:1  $\frac{2}{3}$  of a charging cord is  $\frac{1}{2}$  meter long. How long is the charging cord? (Answer in meters.)
- of a \$50 prize? (2) 8 is 25% of what number? (3) 14 is what percent of 200? (4) Write 6.25% as a decimal, then as a fraction in lowest terms. (5) Find the total cost of a \$16 item after a sales tax of 6.25% is added. (6) A 3% tax on a \$100 item adds \_\_\_\_ dollars to the cost. A 3% tax on a \$1 item adds \_\_\_\_ dollars to the cost.
- The table shows temperatures at the South Pole before and after midnight on October 10–11, 2019.

| Time     | Hours<br>after<br>Midnight | Temp<br>°F |
|----------|----------------------------|------------|
| 8:00 pm  | -4                         | -42        |
| 9:00 pm  | -3                         | -42        |
| 10:00 pm | -2                         | -41        |
| 11:00 pm | -1                         | -40        |
| Midnight | 0                          | -39        |
| 1:00 am  | 1                          | -39        |
| 2:00 am  | 2                          | -38        |



Plot the data on graph paper and label the plot. Describe any patterns you see.

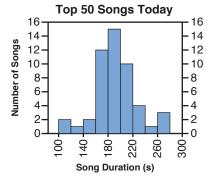
My car drives 570 mi with 15 gal of gas.

(1) Mental math/Pencil and paper (a) If I drive
57 mi, I'll use \_\_\_ gal. (b) If I drive 5,700 mi, I'll
use \_\_\_ gal. (c) If I have 5 gal left, I can drive
\_\_\_ more mi. (d) I can drive \_\_\_ mi with 30
gal. (2) Calculator Calculate both unit rates for
the proportional relationship. (3) (a) If I drive
532 mi, I'll use \_\_\_ gal. (b) If I have 11 gal left, I
can drive \_\_\_ more mi. (4) Make a two-column
table using your answers to (1a), (1c), (1d),
(3a), and (3b). Then use graph paper to plot the
values in the table. Label the axes of your plot.

- 6:5 **(1)** Which of the numbers  $5, -7, \frac{2}{3}, -\frac{1}{2}$  is farthest from 0 on a number line? Which is closest to 0? **(2)** True or False:  $\frac{1}{2} > -8$ . **(3)** Explain why -(-0.2) = 0.2 makes sense.
  - A farmer uses a tractor to plant corn quickly in the springtime. The farmer plants 216 acres every 12 hours. Create a formula for the number

of acres the farmer plants in n hours.

(1) Look up the 50 top songs on a music streaming service. Type each song's duration into a spreadsheet. (2) Write a sentence about the data giving a measure of center and a measure of variability. (3) Make a histogram of the data.\* (4) Write a sentence describing the overall pattern of the distribution and any striking deviations from the overall pattern. (5) Imagine that one year from now, you go back online and repeat (1)–(4). In what ways would you expect the data distribution to look similar? What differences would you expect to see?

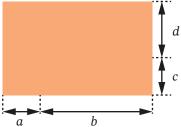


\*Use this histogram for (4) and (5) if you don't do (3).

6:8 *Pencils down* If r = 1.748, what is the value of 0.96r + 0.04r - r?

- 6:9 How much of a  $\frac{3}{4}$ -ton truckload is  $\frac{2}{3}$  ton of gravel?
- 6:10 In the month of February 2021, there were 20 weekdays and 8 weekend days. Here are some questions about that month. (1) (Circle all of the correct answers.) The ratio of weekdays to weekend days was 20:8 10:4 5:2 5:7. (2) There were \_\_\_\_ times as many weekdays as weekend days. (3) True or false:  $\frac{5}{7}$  of the days that month were weekdays. (4) Approximately what percent of the days that month were weekdays?
- 6:11 The diagram shows a rectangle.
  The variables *a*, *b*, *c*, and *d* are lengths in meters.

  (1) Using the



- variables, write three different expressions for the area of the rectangle. (2) Choose two of your expressions and show that they are equivalent by applying properties of operations. (3) State the property or properties you used.
- **6:12 (1)** What is the area of the triangle in the coordinate plane with vertices (1, 2), (-5, 2), and (-8, 9)? **(2)** How does the area change if we change the third vertex to (-3, 9)?
- 6:13 Pencils down Think about the equation  $241p = \frac{3}{4}$ . Is there a whole number that solves it? Is there a non-whole number that solves it? Convince a classmate that your answers are right.
- 6:14 Pencil and paper (1)  $81.53 \div 3.1 = ?$ (2)  $\frac{7}{8} \div \frac{2}{3} = ?$  (3) Check both of your answers by multiplying.

# The 14 Math Milestones™ tasks for grade 6 have been carefully crafted to embody grade 6 mathematics on one page.

| 6:1  | Charging Cord                         | CA  | 6.NS.A.1, 6.EE.B.7 |
|------|---------------------------------------|-----|--------------------|
| 6:2  | Prizes, Prices, and Percents          | СР  | 6.RP.A.3c          |
| 6:3  | South Pole Temperatures               | CA  | 6.NS.C.7, 8        |
| 6:4  | Gas Mileage                           | CA  | 6.RP.A.2, 3        |
| 6:5  | Positive and Negative Numbers         | С   | 6.NS.C.6, 7        |
| 6:6  | Planting Corn                         | С   | 6.RP.A, 6.EE.C.9   |
| 6:7  | Song Length Distribution              | CPA | 6.SP               |
| 6:8  | Evaluating an Expression              | Р   | 6.EE.A             |
| 6:9  | Truckload of Gravel                   | CA  | 6.NS.A.1, 6.EE.B.7 |
| 6:10 | Weekdays and Weekend Days             | С   | 6.RP.A.1           |
| 6:11 | Area Expressions                      | С   | 6.EE.A             |
| 6:12 | Coordinate Triangle                   | СР  | 6.G.A.1, 3         |
| 6:13 | Is There a Solution? (Multiplication) | СР  | 6.EE.B.5           |
| 6:14 | Dividing Decimals and Fractions       | Р   | 6.NS.A.1, 6.NS.B   |

C = Task has a conceptual focus.

P = Task has a procedural skill & fluency focus.

A = Task has an application focus.

#### **Standards for Mathematical Practice**

| MP.1 | Make sense of problems and persevere in solving them.            | 6:4, 6:13                      |
|------|--|--------------------------------|
| MP.2 | Reason abstractly and quantitatively.                            | 6:1, 6:2-4 6:6, 6:9-11         |
| MP.  | Construct viable arguments and critique the reasoning of others. | 6:5, 6:7, 6:11, 6:13           |
| MP.  | Model with mathematics.  | 6:2, 6:4, 6:6, 6:7             |
| MP.  | Use appropriate tools strategically.                             | 6:1, 6:4, 6:7, 6:9             |
| MP.  | Attend to precision.   | 6:4, 6:8, 6:10, 6:14           |
| MP.  | Look for and make use of structure.                              | 6:2, 6:4, 6:5(3), 6:8, 6:11–13 |
| MP.8 | Express regularity in repeated reasoning.                        | 6:4, 6:5(1), 6:6               |

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The cost of a phone is the phone's price, \$264, plus 6.25% tax. **(1)** Use the expression P + 0.0625 \* P to find the cost. (2) Use the expression P \* 1.0625 to find the cost. (3) Apply properties of operations to the expression P +

0.0625 \* P to produce the expression P \* 1.0625.

Pencils down Think about the equation  $x + 4\frac{1}{8} = \frac{2}{3}$ . Is there a positive number that solves it? Is there a negative number that solves it? Tell how you decided.

(1) Calculate. (a) -4.1 + 4 (b)  $5 \div (-6)$ 

(c) -1(-1-1) (d)  $2-(-\frac{1}{2})$  (e)  $(-\frac{3}{8})(-8)$ (f)  $0-\frac{1}{3}$  (g)  $\frac{1}{7.9}*7.9$  (h)  $(\frac{1}{2}-\frac{1}{4})(-9+9)$ .

(2) Show calculation 1(a) on a number line.

A utility pole 24 feet long has  $28\frac{1}{4}$ inch circumference at the top and  $47\frac{1}{9}$ -inch circumference 6 feet from the base. Create and label a scale drawing of the pole in side view, with scale  $\frac{1}{4}$  inch = 1 foot.



distance (mi)

Car A

7:6

Car B distance (mi)

Car A and Car B were moving at constant speed, as shown in the graphs. (1) At the end of the first minute, how many miles had each car moved? (2) Which car was moving faster? (3) For the faster car, write a formula for the number of miles moved in *n* minutes. **(4)** How many miles does the faster car move in 10 minutes?

7:10 In  $\triangle$ ABC, side AB is 4 units long, side BC is 3 units long, and angle A measures 30°. Sketch two ways  $\triangle ABC$  might look.

Nechama is shopping online for a ticket to a play. Website A offers a discount of \$7.50 off the theater price. Website



B offers a discount of 25% off the theater price. (1) Is it mathematically possible that Website A is a better deal than Website B? (2) Is it mathematically possible that Website B is a better deal than Website A? Prove your answers.

In 1972 in Loma, Montana, the temperature changed from  $-54^{\circ}$ F to  $+49^{\circ}$ F in a 24-hr period. Calculate the average rate at which the temperature changed. Answer to the nearest tenth in units of degrees/hr.

7:13 A 15.1-in long wire is bent into the shape of a circle with 2.9 in left over. To the nearest 0.1 in, what is the diameter of the circle?

Rose and Liba both solved this problem: Jannat has 4 packs of balloons and 5 single balloons—29 balloons in all. How many balloons are in a pack? Explain both of Rose's steps. Check that Liba's equations are all true statements about the balloons.

| Rose            | Liba   |
|-----------------|--|
| 29 - 5 = 24     | Let <i>x</i> be the # of balloons in a pack. |
| $24 \div 4 = 6$ | 4x + 5 = 29                                  |
|                 | 4x = 24                                      |
|                 | x = 6  |

Write each sum as a product with the given factor. *Example:*  $8 + 6x = 2 \cdot ?$ Answer: 8 + 6x = 2(4 + 3x). (1)  $6y + 12 = 3 \cdot ?$ (2)  $-5w + 35 = (-5) \cdot ?$  (3)  $4z + 1 = 4 \cdot ?$ **(4)**  $9ay - 9by + 27cy = (9y) \cdot ?$ 

"Foul Play." The Hawks were leading the Pistons in basketball by a score of 100-98. Just as time was running out, a Pistons player tried a 3-point shot.



His defender had two choices: allow the shot, or stop it by fouling the Pistons player. Fouling would give the Pistons player 3 onepoint free throws. The defender chose to foul and later wondered if it was a good choice. (1) To analyze the defender's choice, let's assume that for the Pistons player, every 3-point shot has probability  $\frac{1}{2}$  of going in, and every free throw has probability 90% of going in. (a) If the defender allows the shot, what is the probability that the shot wins the game as time runs out? **(b)** If the defender stops the shot by fouling, estimate the probability that the free throws win the game. (2) Write a paragraph arguing for or against the defender's choice, based on probability calculations and/or simulations.

7:7 If the speed limit in Canada is 100 km/hr and you are driving 65 mph, are you over or under the limit? By how much?

In 2018, an oil company rented an oil rig for \$100,000 per day. The company drilled a well and started pumping oil. (1) How much oil must be sold each day to equal the rental cost? Note: 42 gal of oil could be sold for \$70 in 2018. (2) The company estimates that the profit, *P*, in millions

- of dollars, after pumping oil for D days is P =0.5D - 40. (a) What is the profit after the first day of pumping oil? (b) Make a table of pairs of values (D, P) and graph the ordered pairs.
- (c) How can the company make \$30M of profit?
- (3) An equivalent expression for P is 0.5(D -
- 80). How does the 80 in this expression relate to the company's situation?

# The 14 Math Milestones™ tasks for grade 7 have been carefully crafted to embody grade 7 mathematics on one page.

| 7:1  | Phone Cost                            | СРА | 7.RP.A.3, 7.EE.A              |
|------|---------------------------------------|-----|-------------------------------|
| 7:2  | Utility Pole Scale Drawing            | Α   | 7.G.A.1, 7.G.B.4              |
| 7:3  | Writing Sums as Products              | СР  | 7.EE.A.1                      |
| 7:4  | "Foul Play"                           | CA  | 7.SP.C                        |
| 7:5  | Is There a Solution? (Addition)       | С   | 7.NS.A.1, 7.EE.B.4            |
| 7:6  | Car A and Car B                       | C A | 7.RP.A.2                      |
| 7:7  | Speed Limit                           | Α   | 7.RP.A.1                      |
| 7:8  | Oil Business                          | Α   | 7.RP.A.2b, 7.EE.A.2, 7.EE.B.4 |
| 7:9  | Calculating with Rational Numbers     | СР  | 7.NS.A                        |
| 7:10 | Triangle Conditions                   | С   | 7.G.A.2                       |
| 7:11 | Ticket Offers                         | C A | 7.RP.A.3, 7.EE.B              |
| 7:12 | Temperature Change                    | CA  | 7.RP.A.1, 7.NS.A              |
| 7:13 | Wire Circle                           | Α   | 7.EE.B.4                      |
| 7:14 | Comparing Rose's and Liba's Solutions | С   | 7.EE.B.4                      |

C = Task has a conceptual focus. P = Task has a procedural skill & fluency focus. A = Task has an application focus. = Task is not designed for use with calculators or other technology.

#### **Standards for Mathematical Practice**

| <b>MP.1</b> Make sense of problems and persevere in solving them.     | 7:3, 7:5, 7:8                   |
|---|---------------------------------|
| MP.2 Reason abstractly and quantitatively.                            | 7:1, 7:6, 7:8, 7:11, 7:12, 7:14 |
| MP.3 Construct viable arguments and critique the reasoning of others. | 7:5, 7:10, 7:11                 |
| MP.4 Model with mathematics.  | 7:2, 7:4, 7:6, 7:8, 7:12, 7:13  |
| MP.5 Use appropriate tools strategically.                             | 7:2, 7:4, 7:7, 7:8, 7:10, 7:13  |
| MP.6 Attend to precision.   | 7:2, 7:7, 7:9, 7:10             |
| MP.7 Look for and make use of structure.                              | 7:1, 7:3, 7:5, 7:8, 7:9, 7:14   |
| MP.8 Express regularity in repeated reasoning.                        | 7:1, 7:11                       |

Standards codes refer to www.corestandards.org. One purpose of the codes is that they may allow a task to shed light on the Standards cited for that task. Conversely, reading the cited Standards may suggest opportunities to extend a task or draw out its implications. Finally, Standards codes may also assist with locating relevant sections in curriculum materials, including materials aligned to comparable standards.



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Math Milestones™ tasks are not designed for summative assessment. Used formatively, the tasks can reveal and promote student thinking. Student work on tasks could be collected in student portfolios.

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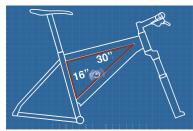
Some Math Milestones  ${}^{\rm I\!M}$  tasks have been designed using image resources from Pixabay.com.



- Xavier's assignment for science class was to write notes to summarize a chapter in his textbook. At 4:45 p.m., he had 12 pages left to summarize. At 6:00 p.m., he had 7 pages left. Assuming a linear model, about how many more hours will it take him to finish summarizing?
- 8:5 Using physical models, transparencies, or geometry software, illustrate the fact that *rotations* take angles to angles of the same measure.
  - Write as a fraction in lowest terms: (1)  $1.041\overline{6}$ .
- A pottery factory has two machines: a fast machine and a slow machine. The fast machine paints a pot in 3 min. The slow machine paints a pot in 10 min. Right now there's a pile of 50 unpainted pots waiting to go into the slow machine, and a pile of 28 unpainted pots waiting to go into the fast machine. (1) If you start the machines at the same time, which machine will finish its pile first? (2) How many min later will the other machine finish its pile? (3) Imagine instead that before starting the machines, you move some unpainted pots from the slow machine's pile to the fast machine's pile. How many pots would you move so that the two machines finish painting at the same time?
- 8:7

  City-to-City Distances & Airline Flight Times

  City-to-city distance (mi) Flight time (hr)
  200 1.0
  300 1.2
  400 1.4
  500 1.6
  - (1) How does flight time between cities depend on city-to-city distance? Answer by creating a function equation that models the data in the table. (2) Use your function to answer: (a) What is the time of flight if two cities are 1,000 mi apart? (b) What is the city-to-city distance if the flight took 2 hr? (3) Use your function and a spreadsheet to extend the table.
- On this blueprint for building a bike, part of the bike is shaped like a right triangle. The longest



side length is illegible because water spilled on the blueprint. Calculate that side length.

8:4 (1) Decide whether each system has exactly one solution, infinitely many solutions, or no solutions. (2) For one system, justify your decision to your classmates in two ways: (a) drawing graphs of solutions; (b) algebraically.

$$\begin{cases} y = \frac{2}{3}x + 1 \\ y = \frac{2}{3}x + 2 \end{cases} \begin{cases} d = 100 - 4t \\ d = 3.5 + t \end{cases} \begin{cases} \frac{1}{8}Q + \frac{3}{8}R = -1 \\ Q + 3R = -8 \end{cases}$$

- A researcher asked people doing exercise to rate their effort level. The researcher also measured people's heart rates. Data were taken on two different days. (1) Use technology to plot the data from both days. (View heart rates in a window from 145 to 175.) Describe the main patterns you see. (2) On one of the days, the exercise room was warm, and on the other day, the room was cool. Which day do you think was the warm day? Tell how you decided, and support your answer with calculations.
- HR, Effort HR, Effort 150.9, 1.3 148.6, 1.6 152.7, 1.9 155.2, 1.5 153.9, 2.3 158.5, 1.8 159.4, 2.1 155.4, 2.9 161.2, 2.1 156.6, 2.9 162.2, 2.3 157.9, 3.1 163.5, 2.4 158.9, 3.6 163.5, 2.7 159.7, 3.7 164.8, 2.7 160.6, 4.1 166.3. 2.9 161.3. 4.2 167.2, 3.0 162.3, 4.3 167.2, 3.3 162.4, 4.6 168.1.3.4 163.4.4.7 169.2, 3.4 164.2, 4.8 169.2, 3.5 164.8, 4.7 165.0.5.0 170.3.3.5 170.8, 3.6 165.4, 5.1 167.0, 5.2 170.4, 3.7 171.9, 3.7 166.5. 5.3 166.7, 5.4 172.3, 3.9 Click here to get the data

online. Each person's heart rate (beats per min.) and effort (1–6

scale) were recorded every 3 min. A group average was then

calculated, creating one data

point such as (150.9, 1.3)

Heart Rate & Effort in

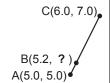
Day 1

Exercise

Day 2

- A chef is cooking soup in a pot. If the chef keeps the soup gently boiling and doesn't cover the pot, water in the soup will evaporate. As water evaporates away, the soup will get thicker and tastier. Let's use a function equation to model the evaporation process: D = 12 - 0.1t. Variable *D* is the depth of the soup in the pot, in units of cm, and variable *t* is the amount of time the soup has been boiling, in units of min. (1) Graph the function. (2a) What is the value of the function for t = 0? **(2b)** What does your value in (2a) refer to in the situation? (2c) How is the situation at t = 0 represented on the graph? (3) What is the value of the slope of the graph, and what is the meaning of that value in the situation? (4) The soup is ready to eat when its depth is  $\frac{2}{3}$  of the initial depth. At what time is the soup ready to eat? (5) Is the model useful for knowing what the depth of the soup would be at time t = 150
- 8:10 Points A, B, and C lie on a straight line in the coordinate plane. By two methods, find the missing vertical coordinate.

min? Why or why not?



- 8:11 Study a proof of the Angle-Angle criterion for triangle similarity. Explain one step of the proof in your own words.
- Design a fish tank that fits into the corner of a room. Use a quarter of a cylinder as a model for the tank. To share your design, make a diagram showing the tank measurements. Also, calculate the weight of the water when your tank is filled (1 m³ of water weighs about 1,000 kg). Write your calculation steps so that a classmate could understand how you did it.

# The 12 Math Milestones™ tasks for grade 8 have been carefully crafted to embody grade 8 mathematics on one page.

| Xavier's Notes                   |   | Α  | 8.F.B.4   |
|----------------------------------|---|--|---|
| Pottery Factory                  |   | PΑ   | 8.EE.C.7b   |
| Bicycle Blueprint                |   | Α  | 8.G.B.7   |
| System Solutions                 |   | СР   | 8.EE.C.8  |
| Rotations Preserve Angle Measure |   | С  | 8.G.A.1   |
| Rational Form                    |   | С  | 8.NS.A.1, 8.EE.A.1  |
| Flight Times and Distances       |   | CA   | 8.F   |
| Heart Rate and Exercise          |   | Α  | 8.SP.A.1-3  |
| Water Evaporation Model          |   | CPA  | 8.F   |
| Missing Coordinate               |   | С  | 8.EE.B  |
| Angle-Angle Similarity Proof     |   | С  | 8.G.A.5   |
| Fish Tank Design                 |   | Α  | 8.G.C.9   |
|                                  | Pottery Factory Bicycle Blueprint System Solutions Rotations Preserve Angle Measure Rational Form | Pottery Factory Bicycle Blueprint System Solutions Rotations Preserve Angle Measure Rational Form Flight Times and Distances Heart Rate and Exercise Water Evaporation Model Missing Coordinate Angle-Angle Similarity Proof | Pottery Factory  Bicycle Blueprint  System Solutions  Rotations Preserve Angle Measure  Rational Form  Flight Times and Distances  Heart Rate and Exercise  Water Evaporation Model  Missing Coordinate  Angle-Angle Similarity Proof |

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#### **Standards for Mathematical Practice**

| 8:1, 8:2, 8:7, 8:10           |
|-------------------------------|
| 8:1, 8:2, 8:7, 8:9            |
| 8:5, 8:8, 8:11                |
| 8:1-3, 8:7-9, 8:12            |
| 8:1, 8:2, 8:4, 8:5, 8:8, 8:12 |
| 8:1, 8:3, 8:6                 |
| 8:4-7, 8:9-11                 |
| 8:2, 8:7                      |
|                               |

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